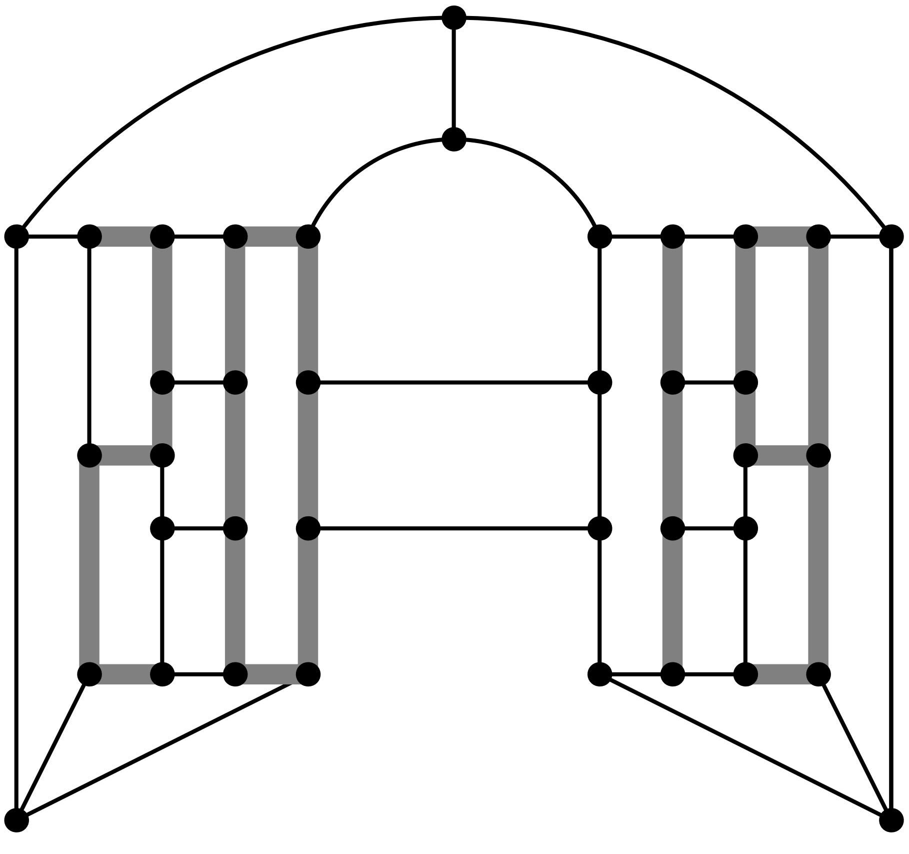
EUROCOMB 2019

10^{th} European Conference

on

Combinatorics, Graph Theory and Applications

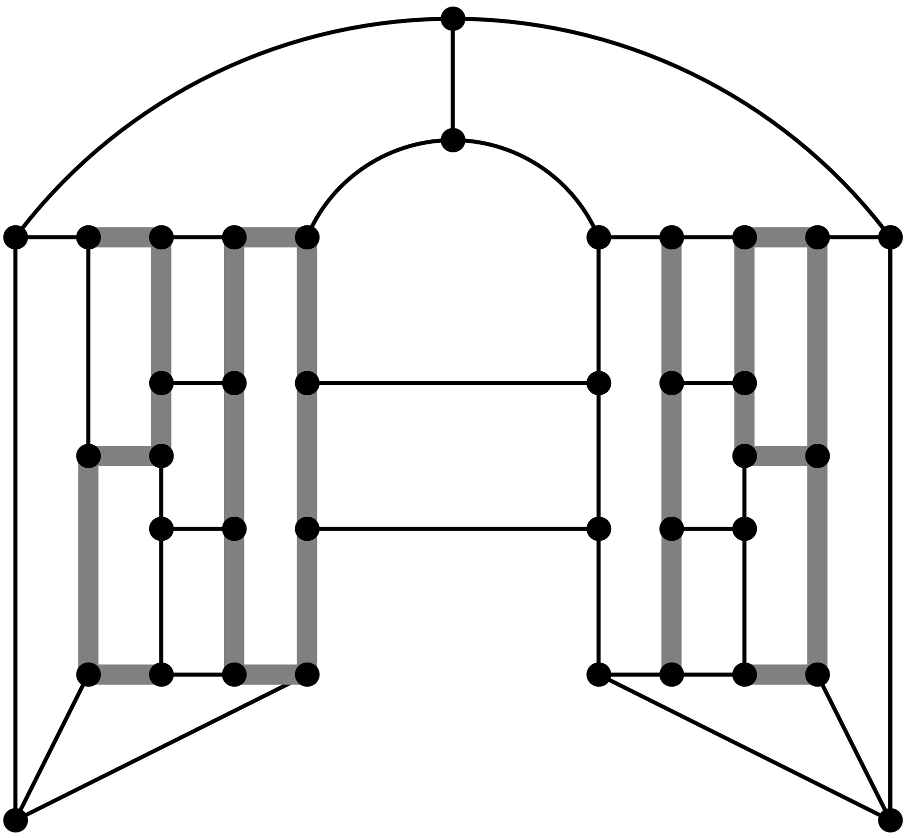
August 26–30, Bratislava, Slovakia



Comenius University in Bratislava
Slovak Mathematical Society

EUROCOMB 2019

Information booklet



Comenius University in Bratislava
Slovak Mathematical Society

Invited speakers:

Andreas Björklund (Sweden)
Marston Conder (New Zealand)
Peter Keevash (UK)
Margaret Readdy (USA)
Alex Scott (UK)
Paul Seymour (USA)
Jozef Širáň (Slovakia)
Hehui Wu (China)
Tamar Ziegler (Israel)

Programme committee:

Jaroslav Nešetřil (co-chair)
Martin Škoviera (co-chair)
Marthe Bonamy
David Conlon
Michael Drmota
Zdenek Dvořák
Stefan Felsner
Ervin Gyori
Lukasz Kowalik
Daniel Kráľ
Daniela Kühn
Keith M. Martin
Bojan Mohar
Dhruv Mubayi
Roman Nedela
Patrice Ossona de Mendez
Edita Pelantová
Marco Pellegrini
Primož Potočnik
Andrzej Ruciński
Oriol Serra
Jozef Skokan
Éric Sopena
Xuding Zhu

Organising committee:

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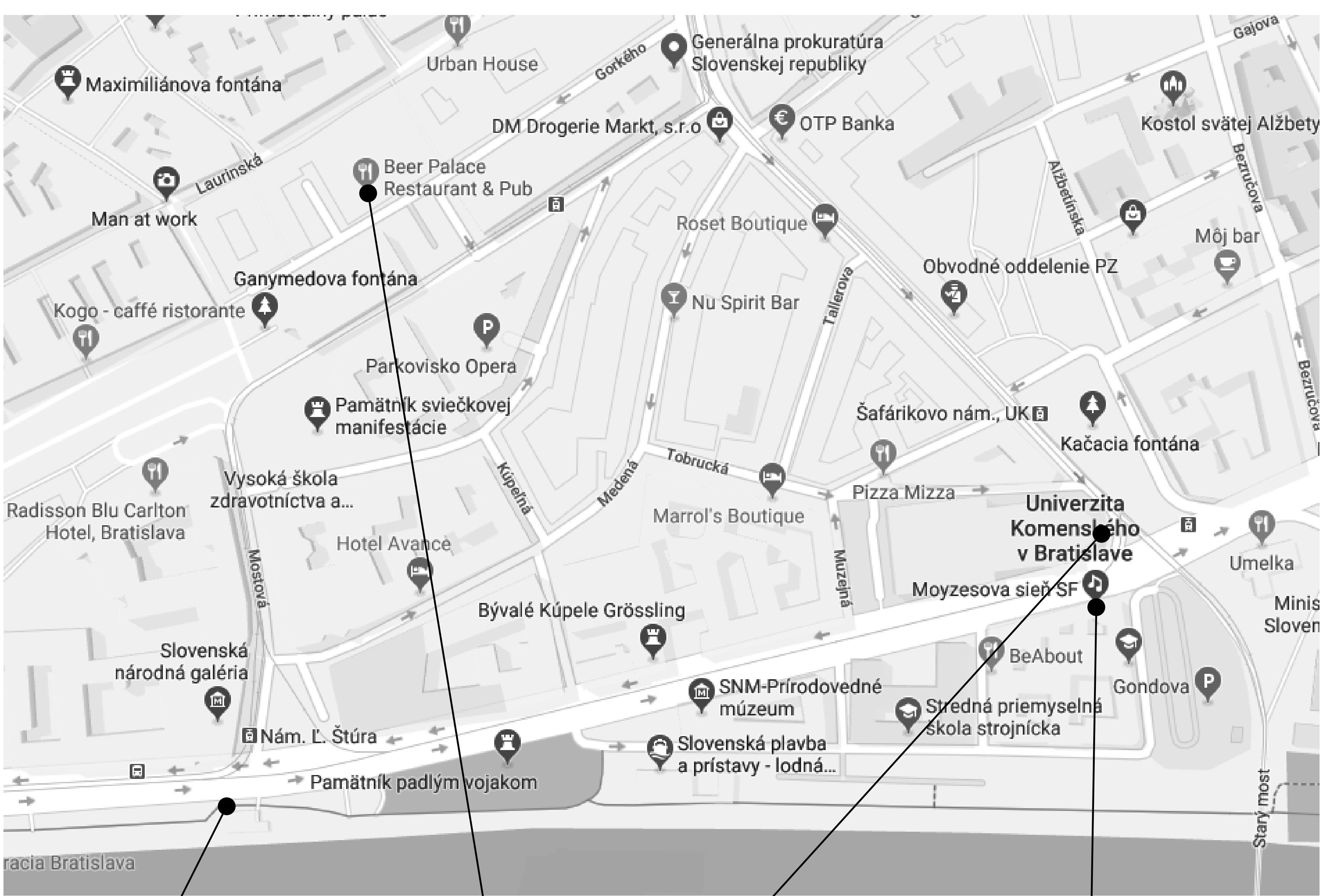
GRAMAD — špičkový výskumný tím UK

Conference logo © Štefan Gyürki 2019

The logo features an important graph discovered by Juraj Bosák around 1965 (also known as the Barnette-Bosák-Lederberg graph). It has 38 vertices and it is the smallest known planar 3-connected non-hamiltonian graph, thus the smallest known counterexample to Tait's Hamiltonian graph conjecture.



Passenger port (Pier 1) Beer Palace (lunches) Venue Moyzesova sieň (conference dinner)

Programme

The short talks are assigned to four rooms: Anton Kotzig Room (No. 2), Štefan Znám Room (No. 3), Juraj Bosák Room (No. 4), Ernest Jucovič Room (Súdna sieň). You can find the detailed assignment of talks on the conference website `http://eurocomb2019.uniba.sk/#programme`.

Monday

09:30 – 10:00	Conference Opening (Aula)
10:00 – 11:00	*P. Seymour: Testing for an odd hole* (Aula)
11:00 – 11:30	coffee break
11:30 – 12:20	2 short talks
12:45 – 14:15	lunch
14:30 – 15:30	*M. Readdy: Geometric proofs of some combinatorial identities of Morel* (Amphitheatre)
15:30 – 16:00	coffee break
16:00 – 17:40	4 short talks

Tuesday

09:00 – 10:00	*J. Širáň: Super-symmetric embeddings of graphs on surfaces: A survey* (Amphitheatre)
10:00 – 10:30	coffee break
10:30 – 12:10	4 short talks
12:45 – 14:15	lunch
14:30 – 15:30	European Prize in Combinatorics award ceremony (Aula)
15:30 – 16:00	coffee break
16:00 – 17:00	*P. Keevash: Isoperimetric stability* (Amphitheatre)
17:00 – 17:50	2 short talks

Wednesday

09:00 – 10:00	*T. Ziegler: Additive combinatorics and algebraic geometry* (Amphitheatre)
10:00 – 10:30	European Prize in Combinatorics lecture (Amphitheatre)
10:30 – 11:00	coffee break
11:00 – 11:50	2 short talks
12:30 – 13:45	lunch
13:55 – 20:00	conference trip

Thursday

09:00 – 10:00	*M. Conder: Edge-transitive graphs and maps* (Amphitheatre)
10:00 – 10:30	European Prize in Combinatorics lecture (Amphitheatre)
10:30 – 11:00	coffee break
11:00 – 12:15	3 short talks
12:45 – 14:15	lunch
14:30 – 15:30	*A. Björklund: Algebraic algorithms for simple paths* (Amphitheatre)
15:30 – 16:00	coffee break
16:00 – 17:40	4 short talks
19:00	conference dinner

Friday

09:00 – 10:00	*A. Scott: Holes in graphs of large chromatic number* (Amphitheatre)
10:00 – 10:30	coffee break
10:30 – 12:10	4 short talks
12:45 – 14:15	lunch
14:30 – 15:30	*H. Wu: Vertex partition with degree constraint* (Amphitheatre)
15:30 – 16:00	coffee break
16:00 – 17:40	4 short talks

Invited lectures

Algebraic algorithms for simple paths

Andreas Björklund

In this talk, we will describe algorithms that detect the presence of a Hamiltonian cycle in bipartite graphs faster than the classical technique of incrementally building a cycle by explicitly bookkeeping what set of vertices have been used so far. We will also look at algorithms for finding a shortest simple cycle through a specified set of vertices, and finding a shortest pair of disjoint paths connecting two given terminal pairs. All of these algorithms follow a common theme:

1. Encode the problem by a multivariate polynomial whose monomials represent solutions to the problem at hand.
2. Find an efficient way of evaluating the polynomial at any given point.
3. Test the polynomial against the zero polynomial.

Edge-transitive graphs and maps

Marston Conder

In this talk I'll describe some recent discoveries about edge-transitive graphs and edge-transitive maps. These are objects that have received relatively little attention compared with their vertex-transitive and arc-transitive siblings.

First I will explain a new approach (taken in joint work with Gabriel Verret) to finding *all edge-transitive graphs of small order* using single and double actions of transitive permutation groups. This has resulted in the determination of all edge-transitive graphs of order up to 47, and all bipartite edge-transitive graphs of order up to 63. (Order 47 is the currently best possible in the non-bipartite case, because the transitive groups of degree 48 are not yet known.) It also led to the answer to a 1967 question by Folkman about the valency-to-order ratio for regular edge- but not vertex-transitive graphs.

Then I'll describe some recent work on *edge-transitive maps*, helped along by workshops at Oaxaca and Banff in 2017. I'll explain how such maps fall into 14 natural classes (two of which are the classes of regular and chiral maps), and how graphs in each class may be constructed and analysed. For example, I'll give the answer to some 18-year-old questions by Širáň, Tucker and Watkins about the existence of particular kinds of such maps on orientable and non-orientable surfaces.

Finally, I'll talk about some new methods for finding *minimum genus embeddings* of graphs on surfaces (developed in joint work with Klara Stokes), with particular emphasis on edge-transitive examples. These have resulted in the answers to some questions dating back to 1988.

Isoperimetric stability

Peter Keevash

Understanding approximate minimisers for isoperimetric problems is of fundamental importance in Combinatorics, Analysis and Geometry. We will discuss recent progress on these problems in the combinatorial setting of the discrete cube (joint with Eoin Long) and the analytic setting of boolean functions with small influence, with applications to sharp thresholds and Extremal Combinatorics via the Junta Method (joint with Noam Lifshitz, Eoin Long and Dor Minzer).

Geometric proofs of some combinatorial identities of Morel

Margaret Readdy

Using the algebraic and geometric combinatorics of the permutahedron, we give proofs of combinatorial identities which arise in the technical heart of Morel's computation of the intersection cohomology of Shimura varieties. No prior background will be assumed. This is joint work with Richard Ehrenborg and Sophie Morel. This is joint work with Richard Ehrenborg and Sophie Morel.

This lecture was sponsored by 365.bank.

Holes in graphs of large chromatic number

Alex Scott

If a graph G has large chromatic number, then what can we say about its induced subgraphs? In particular, if G does not contain a large clique, then what holes must it contain? Thirty years ago, Andras Gyarfas made a sequence of beautiful conjectures on this topic. We will discuss the recent resolution of these conjectures, and other related results. Joint work with Maria Chudnovsky, Paul Seymour and Sophie Spirkl.

Testing for an odd hole

Paul Seymour

There was major progress on perfect graphs in the early 2000's: Chudnovsky, Robertson, Thomas and I proved the "strong perfect graph theorem" that a graph is perfect if and only if it has no odd hole or odd antihole; and Chudnovsky, Cornuejols, Liu, Vuscovic and I found a polynomial-time algorithm to test whether a graph has an odd hole or odd antihole, and thereby test if it is perfect. (A "hole" is an induced cycle of length at least four, and an "antihole" is a hole in the complement graph.)

What we couldn't do then was test whether a graph has an odd hole, and this has stayed open for the last fifteen years, despite some intensive effort. I am happy to report that in fact it can be done in poly-time (in time $O(|G|^9)$, the same as the old algorithm for checking perfection), and in this talk we explain how.

Joint work with Maria Chudnovsky, Alex Scott, and Sophie Spirkl.

Super-symmetric embeddings of graphs on surfaces: A survey

Jozef Širáň

We will be interested in symmetries of cellular embeddings of graphs on closed surfaces. Such embeddings are known as *maps*, and symmetries are a colloquial synonym for *map automorphisms*. The automorphism group of a map is known to act freely on mutually incident vertex-edge-face triples, and on mutually incident vertex-edge pairs in the case of orientation-preserving automorphisms on orientable surfaces. If this action is also transitive, and hence regular, then the map itself is *regular*, or *orientably-regular* in the latter case. From the point of view of automorphism groups, regular and orientably-regular maps exhibit the highest 'level of symmetry' among all maps on general surfaces and orientable surfaces, respectively.

One may, however, consider further embedding transformations, known as *external symmetries*, which are not automorphisms but can make certain regular maps even 'more symmetric'. Examples of such regular maps are those which are, in addition, self-dual and self-Petrie-dual, with the corresponding self-dualities being one type of external symmetries. Such 'totally dual' maps are sometimes said to have *trinity symmetry*.

There is a further way of introducing 'external symmetries' which not as easy to describe and we will do it only for orientably-regular maps to keep things simple. Suppose that one re-embeds a graph underlying an orientably-regular map of valency d in such a way that, at every vertex, instead of the local cyclic ordering of edges emanating from the vertex one takes the j-th power of this ordering for some fixed j coprime to d. This operation may be called *taking the j-th rotational power* of the map. If the new map happens to be isomorphic to the original one, the power j is said to be an *exponent* of the map. The highest level of such external 'rotational power symmetry' occurs if all units j mod d are exponents; such maps are known as *kaleidoscopic*.

In our setting, external symmetries of a regular map will be compositions of self-dualities and rotational powers corresponding to exponents. The 'ultimate level of symmetry' is then achieved by regular maps that are both kaleidoscopic and have trinity symmetry; these deserve to be called *super-symmetric*.

In our talk we will survey results on the existence of regular maps with given valency and face length that are kaleidoscopic, or exhibit trinity symmetry, or are super-symmetric. Constructions of such maps are often tricky and we will give a glimpse into some of the underpinning methods.

This lecture was sponsored by Slovenská sporiteľňa.

Vertex partition with degree constraint

Hehui Wu

A well known result of Stiebitz, which originally was a conjecture of Thomassen, stated that given any natural number s, t, if a graph has minimum degree at least $s + t + 1$, there exists a vertex partition into two parts, such that one part has minimum degree at least s, and the other part has minimum degree at least t. In this talk, we will give a survey of recent results on vertex partition with degree constraint. In particular, with Yan Wang, we proved that for any positive real number s and t, if a graph has average degree at least $s + t + 2$, then there exists a vertex partition into two parts, such that one part has average degree at least s, and the other part has average degree at least t. This answer a question of Csóke, Lo, Norin, Yepremyan and the speaker.

Additive combinatorics and algebraic geometry

Tamar Ziegler

We describe recent connections between questions in additive combinatorics and algebraic geometry of affine varieties defined by few polynomials in many variables. In particular the notion of rank (or strength in the algebraic geometry terminology) plays an important role in many interesting problems in both regimes.

This lecture was sponsored by 365.bank.

Short talks

MB-homogeneous graphs and some new HH-homogeneous graphs

A. Aranda (with D. Hartman)

We present a result showing that any countably infinite HH-homogeneous graph that does not contain the Rado graph as a spanning subgraph has finite independence number; from this we derive a classification of MB-homogeneous graphs. Additionally, we present constructions that yield new HH-homogeneous graphs.

Pop-stack sorting and its image: Permutations with overlapping runs

A. Asinowski (with C. Banderier, S. Billey, B. Hackl, S. Linusson)

Pop-stack sorting is an important variation for sorting permutations via a stack. A single iteration of pop-stack sorting is the transformation $T\colon S_n \to S_n$ that reverses all the maximal descending sequences of letters in a permutation. We investigate structural and enumerative aspects of *pop-stacked permutations* – the permutations that belong to the image of $\mathcal{S}_n$ under T. This work is part of a project aiming to provide the full combinatorial analysis of sorting with a pop-stack, as it was successfully done for sorting with a stack (though, even in this case, some famous problems are still open). The first results already show that pop-stack sorting has a very rich combinatorial structure, and leads to surprising phenomena.

Maximal edge-colorings of graphs

S. Babiński (with A. Grzesik)

For graph G of order n a maximal edge-coloring is a proper partial coloring with $\chi'(K_n)$ colors such that adding any edge to G in any color makes it improper. Meszka and Tyniec proved that for some numbers of edges it is impossible to find such a graph, and provided constructions for some other numbers of edges. However, for many values, the problem remained open. We give a complete solution of this problem for all even values of n and for odd $n \geq 37$.

An improved upper bound on the growth constant of polyiamonds

G. Barequet (with G. Rote, M. Shalah)

A polyiamond is an edge-connected set of cells on the triangular lattice. Let $T(n)$ denote the number of distinct (up to translation) polyiamonds made of n cells. It is known that the sequence $T(n)$ has an asymptotic growth constant, i.e., the limit $\lambda_T := \lim_{n\to\infty} T(n+1)/T(n)$ exists, but the exact value of λ_T is still unknown. In this paper, we improve considerably the best known upper bound on λ_T from 4 to 3.6108.

ε-colorings of strips

F. Bock

A special case of the Hadwiger-Nelson problem is to color a strip instead of the whole plane. The aim is to maximize the width of the strip such that it still permits a coloring with c colors. We present a coloring that improves the recently best known value for 4 colors. This is discovered by considering colorings that satisfy slightly stronger distance conditions. Moreover, we can show under a sensible assumption that this value is best possible for the stronger version of the distance conditions.

Most principal permutation classes, and t-stack sortable permutations, have nonrational generating functions

M. Bóna

We prove that for any fixed n, and for most permutation patterns q, the number $\mathrm{Av}_{n,\ell}(q)$ of q-avoiding permutations of length n that consist of ℓ skew blocks is a monotone decreasing function of ℓ. We then show that this implies that for most patterns q, the generating function $\sum_{n\geq 0} \mathrm{Av}_n(q)z^n$ of the sequence $\mathrm{Av}_n(q)$ of the numbers of q-avoiding permutations is not rational. Placing our results in a broader context, we show that for rational power series $F(z)$ and $G(z)$ with nonnegative real coefficients, the relation $F(z) = 1/(1-G(z))$ is supercritical, while for most permutation patterns q, the corresponding relation is not supercritical.

Cycles of length three and four in tournaments

T. Chan (with A. Grzesik, D. Kráľ, J. A. Noel)

Linial and Morgenstern conjectured that, among all n-vertex tournaments with $d\binom{n}{3}$ cycles of length three, the number of cycles of length four is asymptotically minimized by a random blow-up of a transitive tournament with all but one part of equal size and one smaller part. We prove the conjecture for $d \geq 1/36$ by analyzing the possible spectrum of adjacency matrices of tournaments. We also demonstrate that the family of extremal examples is broader than expected and give its full description for $d \geq 1/16$.

On 2-factors with a specified number of components in line graphs

S. Chiba (with Y. Egawa, J. Fujisawa, A. Saito, I. Schiermeyer, M. Tsugaki, T. Yamashita)

Kaiser and Vrána [European J. Combin. **33** (2012), 924–947] showed that every 5-connected line graph of minimum degree at least 6 is hamiltonian, which gives a partial solution to Thomassen's Conjecture on hamiltonicity of line graphs [J. Graph Theory **10** (1986), 309–324]. In this paper, we prove that every 5-connected line graph of sufficiently large order compared with a given positive integer k and of minimum degree at least 6 also has a 2-factor with exactly k cycles. In order to show this result, we investigate minimum degree conditions for the existence of such a 2-factor in hamiltonian line graphs.

Cohomology groups of non-uniform random simplicial complexes

O. Cooley (with N. Del Giudice, M. Kang, P. Sprüssel)

We consider a model of a random simplicial complex generated by taking the downward-closure of a non-uniform binomial random hypergraph, in which each set of $k+1$ vertices forms an edge with some probability p_k independently, where p_k depends on k and on the number of vertices n. We consider a notion of connectedness on this model according to the vanishing of cohomology groups over an arbitrary abelian group R. We prove that this notion of connectedness displays a phase transition and determine the threshold. We also prove a hitting time result for a natural process interpretation, in which simplices and their downward-closure are added one by one.

Tiling edge-coloured graphs with few monochromatic bounded-degree graphs

J. Corsten (with W. Mendonça)

We prove that for all integers $\Delta, r \geq 2$, there is a constant $C = C(\Delta, r) > 0$ such that the following is true for every sequence $\mathcal{F} = \{F_1, F_2, \ldots\}$ of graphs with $v(F_n) = n$ and $\Delta(F_n) \leq \Delta$ for every $n \in \mathbb{N}$. In every r-edge-coloured K_n, there is a collection of at most C monochromatic copies from $\mathcal{F}$ whose vertex-sets partition $V(K_n)$. This makes progress on a conjecture of Grinshpun and Sárközy.

On the largest component of the critical random digraph

M. Coulson

We consider the largest component of the random digraph $D(n,p)$ inside the critical window $p = n^{-1} + \lambda n^{-4/3}$. We show that the largest component $\mathcal{C}_1$ has size of order $n^{1/3}$ in this range. In particular we give explicit bounds on the probabilities that $|\mathcal{C}_1|n^{-1/3}$ is very large or very small that are analogous to those given by Nachmias and Peres for $G(n,p)$.

Nearly orthogonal vectors and small antipodal spherical codes

C. Cox (with B. Bukh)

How can $d+k$ vectors in $\mathbb{R}^d$ be arranged so that they are as close to orthogonal as possible? In particular, define $\theta(d,k) := \min_X \max_{x \neq y \in X} |\langle x, y\rangle|$ where the minimum is taken over all collections of $d+k$ unit vectors $X \subseteq \mathbb{R}^d$. In this work, we focus on the case where k is fixed and $d \to \infty$. In establishing bounds on $\theta(d,k)$, we find an intimate connection to the existence of systems of $\binom{k+1}{2}$ equiangular lines in $\mathbb{R}^k$. Using this connection, we are able to pin down $\theta(d,k)$ whenever $k \in \{1,2,3,7,23\}$ and establish asymptotics for general k. The main tool is an upper bound on $\mathbb{E}_{x,y\sim\mu} |\langle x, y\rangle|$ whenever μ is an isotropic probability mass on $\mathbb{R}^k$, which may be of independent interest. Our results translate naturally to the analogous question in $\mathbb{C}^d$. In this case, the question relates to the existence of systems of k^2 equiangular lines in $\mathbb{C}^k$, also known as SIC-POVM in physics literature.

Tree Pivot-Minors and Linear Rank-Width

K. K. Dabrowski (with F. Dross, J. Jeong, M. M. Kanté, O-j. Kwon, S-i. Oum, D. Paulusma)

Treewidth and its linear variant path-width play a central role for the graph minor relation. Rank-width and linear rank-width do the same for the graph pivot-minor relation. Robertson and Seymour (1983) proved that for every tree T there exists a constant c_T such that every graph of path-width at least c_T contains T as a minor. Motivated by this result, we examine whether for every tree T there exists a constant d_T such that every graph of linear rank-width at least d_T contains T as a pivot-minor. We show that this is false if T is not a caterpillar, but true if T is the claw.

Maximum number of triangle-free edge colourings with five and six colours

A. Dankovics (with F. Botler, J. Corsten, N. Frankl, H. Hàn, A. Jiménez, J. Skokan)

Let $k \geq 3$ and $r \geq 2$ be natural numbers. For a graph G, let $F(G, k, r)$ denote the number of colourings of the edges of G with colours $1, \ldots, r$ such that, for every colour $c \in \{1, ..., r\}$, the edges of colour c contain no complete graph on k vertices K_k. Let $F(n, k, r)$ denote the maximum of $F(G, k, r)$ over all graphs G on n vertices. The problem of determining $F(n, k, r)$ was first proposed by Erdős and Rothschild in 1974, and has so far been solved only for $r = 2, 3$, and a small number of other cases. In this paper we consider the question for the cases $k = 3$ and $r = 5$ or $r = 6$. We almost exactly determine the value $F(n, 3, 6)$ and approximately determine the value $F(n, 3, 5)$ for large values of n. We also characterise all extremal graphs for $r = 6$ and prove a stability result for $r = 5$.

Symbolic method and directed graph enumeration

S. Dovgal (with É. de Panafieu)

We introduce the arrow product, a systematic generating function technique for directed graph enumeration. It provides short proofs for previous results of Gessel on the number of directed acyclic graphs and of Liskovets, Robinson and Wright on the number of strongly connected directed graphs. We also recover Robinson's enumerative results on directed graphs where all strongly connected components belong to a given family.

Edge colorings avoiding patterns

M. Dębski

We say that a pattern is a graph together with an edge coloring, and a pattern $P = (H, c)$ occurs in some edge coloring c' of G if c', restricted to some subgraph of G isomorphic to H, is equal to c up to renaming the colors. Inspired by Matoušek's visibility blocking problem, we study edge colorings of cliques that avoid certain patterns. We show that for every pattern P, such that the number of edges in P is at least the number of vertices in P plus the number of colors minus 2, there is an edge coloring of K_n that avoids P and uses linear number of colors; the same also holds for finite sets of such patterns.

A rainbow blow-up lemma for almost optimally bounded edge-colourings

S. Ehard (with S. Glock, F. Joos)

A subgraph of an edge-coloured graph is called rainbow if all its edges have different colours. We prove a rainbow version of the blow-up lemma of Komlós, Sárközy and Szemerédi that applies for almost optimally bounded edge-colourings. A corollary of this is that there exists a rainbow copy of any bounded-degree spanning subgraph H in a quasirandom host graph G, assuming that the edge-colouring of G fulfills a boundedness condition that can be seen to be almost best possible. This has many interesting applications beyond rainbow colourings, for example to graph decompositions. There are several well-known conjectures in graph theory concerning tree decompositions, such as Kotzig's conjecture and Ringel's conjecture. We adapt these conjectures to general bounded-degree subgraphs, and provide asymptotic solutions using our result on rainbow embeddings.

Bounding the cop number of a graph by its genus

J. Erde (with N. Bowler, F. Lehner, M. Pitz)

The game of cops and robbers is a pursuit game played on a graph G in which a group of cops tries to catch a robber, where both are allowed to move along to edges of G. The cop number of G, denoted by $c(G)$, is the smallest number of cops needed to catch a robber on G. Schröder showed that $c(G) \leq \lfloor \frac{3}{2} \rfloor g(G) + 3$, where $g(G)$ is the genus of G, that is, the smallest k such that G can be drawn on an orientable surface of genus k. Furthermore, he conjectured that this bound could be improved to $c(G) \leq g(G) + 3$. By relating the game of cops and robbers to a topological game played on a surface we prove that $c(G) \leq \lceil \frac{4}{3} g(G) \rceil + 3$.

Resilience with respect to Hamiltonicity in random graphs

A. Espuny Díaz (with P. Condon, A. Girão, J. Kim, D. Kühn, D. Osthus)

The local resilience of a graph G with respect to a property $\mathcal{P}$ measures how much one has to change G locally in order to destroy $\mathcal{P}$. We prove 'resilience' versions of several classical results about Hamiltonicity for the graph models $G_{n,p}$ and $G_{n,d}$. In the setting of random regular graphs, we prove a resilience version of Dirac's theorem. More precisely, we show that, whenever d is sufficiently large compared to $\varepsilon > 0$, a.a.s. the following holds. Let G' be any subgraph of the random n-vertex d-regular graph $G_{n,d}$ with minimum degree at least $(1/2 + \varepsilon)d$. Then G' is Hamiltonian. This proves a conjecture of Ben-Shimon, Krivelevich and Sudakov. For the binomial random graph $G_{n,p}$, we prove a resilience version of Pósa's Hamiltonicity condition, and show that a natural guess for a resilience version of Chvátal's theorem fails to be true.

Reconfiguration graph for vertex colourings of weakly chordal graphs

C. Feghali (with J. Fiala)

The reconfiguration graph $R_k(G)$ of the k-colourings of a graph G contains as its vertex set the k-colourings of G and two colourings are joined by an edge if they differ in colour on just one vertex of G. We show that for each $k \geq 3$ there is a k-colourable weakly chordal graph G such that $R_{k+1}(G)$ is disconnected. We also introduce a subclass of k-colourable weakly chordal graphs which we call k-colourable compact graphs and show that for each k-colourable compact graph G on n vertices, $R_{k+1}(G)$ has diameter $O(n^2)$. We show that this class contains all k-colourable co-chordal graphs and when $k = 3$ all 3-colourable $(P_5, \overline{P_5}, C_5)$-free graphs. We also mention some open problems.

A note on covering Young diagrams with applications to local dimension of posets

S. Felsner (with T. Ueckerdt)

We prove that in every cover of a Young diagram with $\binom{2k}{k}$ steps with generalized rectangles there is a row or a column in the diagram that is used by at least $k+1$ rectangles. We show that this is best-possible by partitioning any Young diagram with $\binom{2k}{k} - 1$ steps into actual rectangles, each row and each column used by at most k rectangles. This answers two questions asked in [Kim J., Martin R. R., Masařík T., Shull W., On difference graphs and the local dimension of posets, arXiv:1803.08641]. Our results can be rephrased in terms of local covering numbers of difference graphs with complete bipartite graphs, which has applications in the recent notion of local dimension of partially ordered sets.

Spectra and eigenspaces of arbitrary lifts of graphs

M. À. Fiol (with C. Dalfó, S. Pavlíková, J. Širáň)

We describe, in a very explicit way, a method for determining the spectra and bases of all the corresponding eigenspaces of arbitrary lifts of graphs (regular or not).

Doubly biased Walker-Breaker games

J. Forcan (with M. Mikalački)

We study doubly biased Walker-Breaker games, played on the edge set of a complete graph on n vertices, K_n. Walker-Breaker game is a variant of Maker-Breaker game, where Walker, playing the role of Maker, must choose her edges according to a walk, while Breaker has no restrictions on choosing his edges. Here we show that for $b \leq \frac{n}{10 \ln n}$, playing a $(2 : b)$ game on $E(K_n)$, Walker can create a graph containing a spanning tree.

Also, we determine a constant $c > 0$ such that Walker has a strategy to make a Hamilton cycle of K_n in the $(2 : \frac{cn}{\ln n})$ game.

Nearly k-distance sets

N. Frankl (with A. Kupavskii)

We say that $S \subset \mathbb{R}^d$ is an *ε-nearly k-distance set* if there exist $1 \leq t_1 \leq \cdots \leq t_k$ such that the distance between any two distinct points of S falls into $[t_1, t_1 + \varepsilon] \cup \cdots \cup [t_k, t_k + \varepsilon]$. In this abstract, we propose to study the quantity $M_k(d) := \lim_{\varepsilon \to 0} \max\{|S| : S$ is an ε-nearly k-distance set in $\mathbb{R}^d\}$. Let $m_k(d)$ be the maximal cardinality of a k-distance set in $\mathbb{R}^d$. We show that $M_k(d) = m_k(d)$ if either $d \geq d(k)$ or $k \leq 3$. We also address a closely related Turán-type problem, studied by Erdős, Makai, Pach, and Spencer in the 80's: given n points in $\mathbb{R}^d$, how many pairs out of them form a distance that belongs to $[t_1, t_1 + 1] \cup \cdots \cup [t_k, t_k + 1]$, where $t_1, \ldots, t_k$ are fixed and any two points in the set are at distance at least 1 apart? We obtain an exact answer for the same k, d as above.

Theory of limits of sequences of Latin squares

F. Garbe (with R. Hancock, J. Hladký, M. Sharifzadeh)

We build up a limit theory for sequences of Latin squares, which parallels the theory of limits of dense graph sequences. Our limit objects, which we call Latinons, are certain two variable functions whose values are probability distributions on $[0, 1]$. Left-convergence is defined using densities of $k \times k$ subpatterns in finite Latin squares, which extends to Latinons. We also provide counterparts to the cut distance, and prove a counting lemma, and an inverse counting lemma.

Guarding isometric subgraphs and Lazy Cops and Robbers

S. González Hermosillo de la Maza (with B. Mohar)

In the game of Cops and Robbers, one of the most useful results is that an isometric path in a graph can be guarded by one cop. In this paper, we introduce the concept of wide shadow on a graph, and use it to provide a short proof of the characterization of 1-guardable graphs. As an application, we show that 3 cops can capture a robber in any planar graph with the added restriction that at most two cops can move simultaneously, proving a conjecture of Yang and strenghtening a classical result by Aigner and Fromme.

A step towards the $3k-4$ conjecture in $\mathbb{Z}/p\mathbb{Z}$ and an application to m-sum-free sets

D. González-Sánchez (with P. Candela, D. J. Grynkiewicz)

The $3k-4$ conjecture in $\mathbb{Z}/p\mathbb{Z}$ states that if A is a nonempty subset of $\mathbb{Z}/p\mathbb{Z}$ satisfying $2A \neq \mathbb{Z}/p\mathbb{Z}$ and $|2A| = 2|A| + r \leq \min\{3|A|-4,\ p-r-3\}$, then A is covered by an arithmetic progression of size at most $|A|+r+1$. In this paper we summarize progress made towards this conjecture in a recent joint paper of the same authors. In that paper we prove first that if $|2A| \leq (2+\alpha)|A| - 3$ for $\alpha \approx 0.136861$ and $|2A| \leq 3p/4$, then A is efficiently covered by an arithmetic progression, as in the conclusion of the conjecture. With a refined argument we prove that we can go up to $\alpha = (\sqrt{33}-5)/4 + o_{|A|,p\to\infty}(1)$ at the cost of restricting $|A| \leq (p-r)/3$. We then use this to investigate the maximum size of m-sum-free sets for $m \geq 3$, i.e., sets $A \subseteq \mathbb{Z}/p\mathbb{Z}$ such that the equation $x+y = mz$ has no solution in A. We obtain that for m fixed, $\lim_{p\to\infty} \max\{|A|/p : A \subseteq \mathbb{Z}/p\mathbb{Z}\ m\text{-sum-free}\} \leq 1/3.1955$ (previously, the best known upper bound was $1/3.0001$).

Majority coloring of infinite digraphs

J. Grytczuk (with M. Anholcer, B. Bosek)

Let D be a finite or infinite digraph. A *majority coloring* of D is a vertex coloring such that at least half of the out-neighbors of every vertex v have different color than v. Let $\mu(D)$ denote the least number of colors needed for a majority coloring of D. It is known that $\mu(D) \leq 4$ for any finite digraph D, and $\mu(D) \leq 2$ if D is acyclic. We prove that $\mu(D) \leq 5$ for any countably infinite digraph D, and $\mu(D) \leq 3$ if D does not contain finite directed cycles. We also state a twin supposition to the famous Unfriendly Partition Conjecture.

Recent developments on unavoidable patterns in 2-colorings of the complete graph

A. Hansberg (with Y. Caro, A. Montejano)

In this manuscript, we review recent developments concerning unavoidable patterns in 2-edge colorings of the complete graph.

Homomorphism-homogeneity classes of countable L-colored graphs

D. Hartman (with A. Aranda)

The notion of homomorphism-homogeneity, introduced by Cameron and Nešetřil, originated as a variation on ultrahomogeneity. By fixing the type of finite homomorphism and global extension, several homogeneity classes, called morphism extension classes, can be defined. These classes are studied for various languages and axiom sets. Hartman, Hubička and Mašulović showed for finite undirected L-colored graphs without loops, where colors for vertices and edges are chosen from a partially ordered set L, that when L is a linear order, the classes HH and MH of L-colored graphs coincide, contributing thus to a question of Cameron and Nešetřil. They also showed that the same is true for vertex-uniform finite L-colored graphs when L is a diamond. In this work, we extend their results to countably infinite L-colored graphs, proving that the classes MH and HH coincide if and only if L is a linear order.

Minimum degree conditions for powers of cycles and paths

E. K. Hng

The study of conditions on vertex degrees in a host graph G for the appearance of a target graph H is a major theme in extremal graph theory. The k^{th} power of a graph F is obtained from F by joining any two vertices at distance at most k. We study minimum degree conditions under which a graph G contains the k^{th} power of cycles and paths of arbitrary specified lengths. We determine precise thresholds, assuming that the order of G is large. This extends a result of Allen, Böttcher and Hladký concerning the containment of squared paths and squared cycles of arbitrary specified lengths and settles a conjecture of theirs in the affirmative.

Big Ramsey degrees of 3-uniform hypergraphs

J. Hubička (with M. Balko, D. Chodounský, M. Konečný, L. Vena)

Given a countably infinite hypergraph $\mathcal{R}$ and a finite hypergraph $\mathcal{A}$, the *big Ramsey degree* of $\mathcal{A}$ in $\mathcal{R}$ is the least number L such that, for every finite k and every k-colouring of the embeddings of $\mathcal{A}$ to $\mathcal{R}$, there exists an embedding f from $\mathcal{R}$ to $\mathcal{R}$ such that all the embeddings of $\mathcal{A}$ to the image $f(\mathcal{R})$ have at most L different colours. We describe the big Ramsey degrees of the random countably infinite 3-uniform hypergraph, thereby solving a question of Sauer. We also give a new presentation of the results of Devlin and Sauer on, respectively, big Ramsey degrees of the order of the rationals and the countably infinite random graph. Our techniques generalise (in a natural way) to relational structures and give new examples of Ramsey structures (a concept recently introduced by Zucker with applications to topological dynamics).

Bijections for generalized Tamari intervals via orientations

A. Humbert (with É. Fusy)

We introduce two bijections for generalized Tamari intervals, which were recently introduced by Préville-Ratelle and Viennot, and proved to be in bijection with rooted non-separable maps by Fang and Préville-Ratelle. Our first construction proceeds via separating decompositions on quadrangulations and can be seen as an extension of the Bernardi-Bonichon bijection between Tamari intervals and minimal Schnyder woods. Our second construction directly exploits the Bernardi-Bonichon bijection and the point of view of generalized Tamari intervals as a special case of classical Tamari intervals (synchronized Tamari intervals); it yields a trivariate generating function expression that interpolates between generalized Tamari intervals and classical Tamari intervals.

Dushnik-Miller dimension of stair contact complexes

L. Isenmann (with D. Gonçalves)

The theorem of Schnyder asserts that a graph is planar if and only if the Dushnik-Miller dimension of its poset of incidence is at most 3. Trotter aksed how this can be generalized to higher dimensions. Towards this goal, Dushnik-Miller dimension has been studied in terms of TD-Delaunay complexes, in terms of orthogonal surfaces, and in terms of polytopes. Here we consider the relation between the Dushnik-Miller dimension and contact systems of stairs in $\mathbb{R}^d$. We propose two different definitions of stairs in $\mathbb{R}^d$ which are connected to the Dushnik-Miller dimension as follows. The first definition allows us to characterize supremum sections, which are simplicial complexes related to the Dushnik-Miller dimension, in two different ways. The second definition provides for any Dushnik-Miller dimension at most $d+1$ complex a representation as a contact system of stairs in $\mathbb{R}^d$.

Extending partial automorphisms of n-partite tournaments

C. Jahel (with J. Hubička, M. Konečný, M. Sabok)

We prove that for every $n \geq 2$ the class of all finite n-partite tournaments (orientations of complete n-partite graphs) has the extension property for partial automorphisms, that is, for every finite n-partite tournament G there is a finite n-partite tournament H such that every isomorphism of induced subgraphs of G extends to an automorphism of H. Our constructions are purely combinatorial (whereas many earlier EPPA results use deep results from group theory) and extend to other classes such as the class of all finite semi-generic tournaments.

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k-hypergraphs with regular automorphism groups

T. Jajcayová (with R. Jajcay)

Regular representations of finite groups, as introduced by Cayley, are among the most natural permutation representations of finite groups. Thus, the question which regular representations appear as full automorphism groups of combinatorial structures has been addressed and resolved for several classes of structures, notably for graphs (where they are called Graphical Regular Representations, GRR's), digraphs (Digraphical Regular Representations, DRR's) as well as for hypergraphs allowing for hyperedges of varying sizes. In the present paper, we focus on k-hypergraphs, which are hypergraphs in which all hyperedges are of the same size k, and address the question which k-regular hypergraphs possess full automorphism groups acting regularly on the vertices. We rely on the concept of a Cayley hypergraph (defined here) and show that all sufficiently large finite groups admit a regular representation as the full automorphism group of a 3-hypergraph.

Edge-coloring of plane graphs with many colors on faces

S. Jendroľ (with J. Czap, J. Valiska)

For a fixed positive integer p, a coloring of the edges of a multigraph G is called p-acyclic coloring if every cycle C in G contains at least $\min\{|C|, p+1\}$ colors. The least number of colors needed for a p-acyclic coloring of G is the p-arboricity of G. This type of coloring was introduced by Nešetřil, Ossona de Mendez, and Zhu in 2014. From a result of Bartnicki et al. (2019) it follows that there are planar graphs with unbounded p-arboricity. In this note we improve a result of Bartnicki et al. on p-arboricity of planar graphs with large girth. In addition, we relax the definition of p-arboricity for plane multigraphs in sense that the requirement is not for all cycles but only for facial ones, and we show that the smallest number of colors needed for such a coloring is a constant (depending on p only).

Multiple list colouring of triangle free planar graphs

Y. Jiang (with X. Zhu)

Colouring of triangle free planar graphs has been studied extensively in the literature. It was proved by Grötzsch that every triangle free planar graph is 3-colourable. On the other hand, Voigt showed that there are triangle free planar graphs that are not 3-choosable. What we are interested in is multiple list colouring of triangle free planar graphs. We prove that for each positive integer m, there is a triangle free planar graph G which is not $(3m + \lceil \frac{m}{17} \rceil - 1, m)$-choosable.

Automorphism groups of edge-transitive maps

G. A. Jones

For each of the 14 classes of edge-transitive maps described by Graver and Watkins, necessary and sufficient conditions are given for a group to be the automorphism group of a map, or of an orientable map without boundary, in that class. Extending earlier results of Širáň, Tucker and Watkins, these are used to determine which symmetric groups S_n can arise in this way for each class. Similar results are obtained for all finite simple groups, building on work of Leemans and Liebeck, Nuzhin and others on generating sets for such groups. It is also shown that each edge-transitive class realises finite groups of every sufficiently large nilpotence class or derived length, and also realises uncountably many non-isomorphic infinite groups.

Resolution of the Oberwolfach problem

F. Joos (with S. Glock, J. Kim, D. Kühn, D. Osthus)

The Oberwolfach problem, posed by Ringel in 1967, asks for a decomposition of K_{2n+1} into edge-disjoint copies of a given 2-factor. We show that this can be achieved for all large n. We actually prove a significantly more general result, which allows for decompositions into more general types of factors. In particular, this also resolves the Hamilton-Waterloo problem for large n.

A new lower bound on Hadwiger-Debrunner numbers in the plane

C. Keller (with S. Smorodinsky)

A family of sets $\mathcal{F}$ is said to satisfy the (p,q)-property if among any p sets in $\mathcal{F}$ some q have a non-empty intersection. Hadwiger and Debrunner (1957) conjectured that for any $p \geq q \geq d+1$ there exists $c = c_d(p,q)$, such that any family of compact convex sets in $\mathcal{R}^d$ that satisfies the (p,q)-property can be pierced by at most c points. In a celebrated result from 1992, Alon and Kleitman proved the conjecture. However, obtaining sharp bounds on $c_d(p,q)$, known as the 'the Hadwiger-Debrunner numbers,' is still a major open problem in combinatorial geometry. The best currently known lower bound on the Hadwiger-Debrunner numbers in the plane is $c_2(p,q) = \Omega(\frac{p}{q}\log(\frac{p}{q}))$, while the best known upper bound is $O(p^{(1.5+\delta)(1+\frac{1}{q-2})})$. In this paper we improve the lower bound significantly by showing that $c_2(p,q) \geq p^{1+\Omega(1/q)}$. Furthermore, the bound is obtained by a family of lines and is tight for all families that have a bounded VC-dimension. Unlike previous bounds on the Hadwiger-Debrunner numbers, which mainly used the weak epsilon-net theorem, our bound stems from a surprising connection of the (p,q)-problem to an old problem of Erdős on points in general position in the plane. We use a novel construction for the Erdős' problem, obtained recently by Balogh and Solymosi using the *hypergraph container method*, to get the lower bound on $c_2(p,3)$. We then generalize the bound to $c_2(p,q)$ for any $q \geq 3$.

Coloring hypergraphs defined by stabbed pseudo-disks and $ABAB$-free hypergraphs

B. Keszegh (with E. Ackerman, D. Pálvölgyi)

What is the minimum number of colors that always suffice to color every planar set of points such that any disk that contains enough points contains two points of different colors? It is known that the answer to this question is either three or four. We show that three colors always suffice if the condition must be satisfied only by disks that contain a fixed point. Our result also holds, and is even tight, when instead of disks we consider their topological generalization, namely *pseudo-disks*, with a non-empty intersection. Our solution uses the equivalence that a hypergraph can be realized by stabbed pseudo-disks if and only if it is *ABAB-free*. These hypergraphs are defined in a purely abstract, combinatorial way and our proof that they are 3-chromatic is also combinatorial.

On the maximum number of odd cycles in graphs without smaller odd cycles

B. Kielak (with A. Grzesik)

We prove that for each odd integer $k \geq 7$, every graph on n vertices without odd cycles of length less than k contains at most $(n/k)^k$ cycles of length k. This generalizes the previous results on the maximum number of pentagons in triangle-free graphs, conjectured by Erdős in 1984, and asymptotically determines the generalized Turán number $\mathrm{ex}(n, C_k, C_{k-2})$ for odd k. In contrast to the previous results on the pentagon case, our proof is not computer-assisted.

Bounding the tripartite-circle crossing number of complete tripartite graphs

R. Kirsch (with C. A. Camacho, S. Fernández-Merchant, L. Kleist, E. Bailey Matson, M. Jelić Milutinović, J. White)

A tripartite-circle drawing of the complete tripartite graph $K_{m,n,p}$ is a drawing in the plane, where each part of the vertex partition is placed on one of three disjoint circles, and the edges do not cross the circles. We present upper and lower bounds on the minimum number of crossings in tripartite-circle drawings of $K_{m,n,p}$ and the exact value for $K_{2,2,n}$. In contrast to 1- and 2-circle drawings which may attain the Harary-Hill bound, our results imply that optimal drawings of the complete graph do not contain balanced 3-circle drawings as subdrawings that do not cross any of the remaining edges.

Sharp bounds for the chromatic number of random Kneser graphs

S. Kiselev (with A. Kupavskii)

Given positive integers $n \geq 2k$, a *Kneser graph* $KG_{n,k}$ is a graph whose vertex set is the collection of all k-element subsets of the set $\{1, \ldots, n\}$, with edges connecting pairs of disjoint sets. A famous result due to L. Lovász states that the chromatic number of $KG_{n,k}$ is equal to $n - 2k + 2$. In this paper, we study the *random Kneser graph* $KG_{n,k}(p)$, obtained from $KG_{n,k}$ by including each of the edges of $KG_{n,k}$ independently and with probability p. We prove that, for any fixed $k \geq 3$, $\chi(KG_{n,k}(1/2)) = n - \Theta(\sqrt[2k-2]{\log_2 n})$. We also provide new bounds for the case of growing k. This significantly improves previous results on the subject, obtained by Kupavskii and by Alishahi and Hajiabolhassan. We also discuss an interesting connection to an extremal problem on embeddability of complexes.

On the Graovac-Pisanski index of a graph

M. Knor (with R. Škrekovski, A. Tepeh)

Let G be a graph. Its Graovac-Pisanski index is

$$\mathrm{GP}(G) = \frac{|V(G)|}{2|\mathrm{Aut}(G)|} \sum_{u \in V(G)} \sum_{\alpha \in \mathrm{Aut}(G)} \mathrm{dist}(u, \alpha(u)),$$

where $\mathrm{Aut}(G)$ is the group of automorphisms of G, and its Wiener index, $W(G)$, is the sum of all distances in G. In the class of trees (unicyclic graphs) on n vertices we find those with the maximum value of Graovac-Pisanski index. We show that the inequality $\mathrm{GP}(G) \leq W(G)$ is not true in general, but it is true for trees.

Flow number and circular flow number of signed cubic graphs

A. Kompišová (with E. Máčajová)

Let $\Phi(G,\sigma)$ and $\Phi_c(G,\sigma)$ denote the flow number and the circular flow number of a flow-admissible signed graph (G,σ), respectively. It is known that $\Phi(G) = \lceil \Phi_c(G) \rceil$ for every unsigned graph G. Based on this fact Raspaud and Zhu in 2011 conjectured that $\Phi(G,\sigma) - \Phi_c(G,\sigma) < 1$ holds also for every flow-admissible signed graph (G,σ). This conjecture was disproved by Schubert and Steffen using graphs with bridges and vertices of large degree. In this paper we focus on cubic graphs, since they play a crucial role in many open problems in graph theory. For cubic graphs we show that $\Phi(G,\sigma) = 3$ if and only if $\Phi_c(G,\sigma) = 3$ and if $\Phi(G,\sigma) \in \{4,5\}$, then $4 \leq \Phi_c(G,\sigma) \leq \Phi(G,\sigma)$. We also prove that all pairs of flow number and circular flow number that fulfill these conditions can be achieved in the family of bridgeless cubic graphs and thereby disprove the conjecture of Raspaud and Zhu even for bridgeless cubic signed graphs. Finally, we prove that all currently known graphs without nowhere-zero 5-flow have flow number and circular flow number 6 and propose several conjectures in this area.

Ramsey properties of edge-labelled graphs via completions

M. Konečný (with J. Hubička, J. Nešetřil)

Motivated by applications in structural Ramsey theory, we describe "metric-like" classes of edge-labelled graphs, study their completion problems and find Ramsey expansions. They turn out to be general enough to incorporate most of the known Ramsey results for edge-labelled graphs under a common framework and also solve a problem of Conant on generalised metric spaces. As a corollary of understanding completions, one obtains homomorphism dualities for these classes.

On the achievable average degrees in 2-crossing-critical graphs

M. Korbela (with P. Hliněný)

c-Crossing-critical graphs are the minimal graphs requiring at least c edge crossings in every drawing in the plane. The structure of these obstructions is very rich for every $c \geq 2$. Although, at least in the first nontrivial case of $c = 2$, their structure is well understood. For example, we know that, aside of finitely many small exceptions, the 2-crossing-critical graphs have vertex degrees from the set $\{3, 4, 5, 6\}$ and their average degree can achieve exactly all rational values from the interval $[3\frac{1}{2}, 4\frac{2}{3}]$. Continuing in depth in this research direction, we determine which average degrees of 2-crossing-critical graphs are possible if we restrict their vertex degrees to proper subsets of $\{3, 4, 5, 6\}$. In particular, we identify the (surprising) subcases in which, by number-theoretical reasons, the achievable average degrees form discontinuous sets of rationals.

Characterization of generalised Petersen graphs that are Kronecker covers

M. Krnc (with T. Pisanski)

The family of generalised Petersen graphs $G(n, k)$, introduced by Coxeter (1950) and named by Watkins (1969) is a family of cubic graphs formed by connecting the vertices of a regular polygon to the corresponding vertices of a star polygon. The Kronecker cover $\mathrm{KC}(G)$ of a simple undirected graph G is a special type of bipartite covering graph of G, isomorphic to the direct (tensor) product of G and K_2. We characterize all generalised Petersen graphs that are Kronecker covers, and describe the structure of their respective quotients. We observe that some of such quotients are again generalised Petersen graphs, and describe all such pairs.

Some results around the Erdős Matching Conjecture

A. Kupavskii (with P. Frankl)

More than 50 years ago, Erdős asked the following question: what is the largest family of k-element subsets of $[n]$ with no s pairwise disjoint sets? In this abstract, we discuss recent progress on this problem and its generalizations.

Ramsey upper density of infinite graphs

A. Lamaison

Let H be an infinite graph. In a two-coloring of the edges of the complete graph on the natural numbers, what is the densest monochromatic subgraph isomorphic to H that we are guaranteed to find? We measure the density of a subgraph by the upper density of its vertex set. This question, in the particular case of the infinite path, was introduced by Erdős and Galvin. Following a recent result for the infinite path, we present bounds on the maximum density for other choices of H, including exact values for a wide class of bipartite graphs.

Asymptotically good local list edge colourings

R. Lang (with M. Bonamy, M. Delcourt, L. Postle)

We study list edge colourings under local conditions. Our main result is an analogue of Kahn's theorem in this setting. More precisely, we show that, for a simple graph G with sufficiently large maximum degree Δ and minimum degree $\delta \geq \ln^{25} \Delta$, the following holds. Suppose that lists of colours $L(e)$ are assigned to the edges of G, such that, for each edge $e = uv$,

$$|L(e)| \geq (1 + o(1)) \cdot \max\{\deg(u), \deg(v)\}.$$

Then there is an L-edge-colouring of G. We also provide extensions of this result for hypergraphs and correspondence colourings, a generalization of list colouring.

Distinguishing tournaments with small label classes

A. Lozano

A *d-distinguishing vertex (arc) labeling* of a digraph is a vertex (arc) labeling using d labels that is not preserved by any nontrivial automorphism. Let $\rho(T)$ ($\rho'(T)$) be the minimum size of a label class in a 2-distinguishing vertex (arc) labeling of a tournament T. Gluck's Theorem implies that $\rho(T) \leq \lfloor n/2 \rfloor$ for any tournament T of order n. We construct a family of tournaments $\mathcal{H}$ such that $\rho(T) \geq \lfloor n/2 \rfloor$ for any tournament of order n in $\mathcal{H}$. Additionally, we prove that $\rho'(T) \leq \lfloor 7n/36 \rfloor + 3$ for any tournament T of order n and $\rho'(T) \geq \lceil n/6 \rceil$ when $T \in \mathcal{H}$ and has order n. These results answer some open questions stated by Boutin.

Permutation snarks of order 2 (mod 8)

E. Máčajová (with M. Škoviera)

A permutation snark is a cubic graph which has a 2-factor consisting of two chordless cycles and is not 3-edge-colourable. Every permutation snark is cyclically 4-edge-connected, has girth at least 5, and its order is twice an odd number. Employing exhaustive computer search, Brinkmann et al. (2013) discovered a cyclically 5-edge-connected permutation snark of order 34, disproving a conjecture of C.-Q. Zhang (1997) that the Petersen graph is the only such graph. Hägglund and Hoffmann-Ostenhof (2017) extended this example to an infinite series of cyclically 5-edge-connected permutation snarks of order $n = 24k + 10$ for every positive integer k. Here we present three general methods of constructing permutation snarks and with their help provide permutation snarks with cyclic connectivity 4 and 5 for every possible order $2 \pmod 8$.

Powers of Hamiltonian cycles in μ-inseparable graphs

G. S. Maesaka (with O. Ebsen, C. Reiher, M. Schacht, B. Schülke)

We consider sufficient conditions for the existence of k-th powers of Hamiltonian cycles in n-vertex graphs G with minimum degree μn for arbitrarily small $\mu > 0$. About 20 years ago Komlós, Sárközy, and Szemerédi resolved the conjectures of Pósa and Seymour and obtained optimal minimum degree conditions for this problem by showing that $\mu = \frac{k}{k+1}$ suffices for large n. Consequently, for smaller values of μ the given graph G must satisfy additional assumptions. We show that inducing subgraphs of density $d > 0$ on linear subsets of vertices and being inseparable, in the sense that every cut has density at least $\mu > 0$, are sufficient assumptions for this problem. This generalises a recent result of Staden and Treglown.

Flexibility of planar graphs without 4-cycles

T. Masařík

Proper graph coloring assigns different colors to adjacent vertices of the graph. Usually, the number of colors is fixed or as small as possible. Consider applications (e.g. variants of scheduling) where colors represent limited resources and graph represents conflicts, i.e., two adjacent vertices cannot obtain the same resource. In such applications, it is common that some vertices have preferred resource(s). However, unfortunately, it is not usually possible to satisfy all such preferences. The notion called flexibility was recently defined in [Dvořák Z., Norin S. and Postle L.,*List coloring with requests*, J. Graph Theory (2019), 1–16]. There instead of satisfying all the preferences the aim is to satisfy at least a constant fraction of the request. Recently, the structural properties of planar graphs in terms of flexibility were investigated. We continue this line of research.Let G be a planar graph with a list assignment L. Suppose a preferred color is given for some of the vertices. We prove that if G is a planar graph without 4-cycles and all lists have size at least five, then there exists an L-coloring respecting at least a constant fraction of the preferences.

On relaxed Šoltés's problem

J. Maxová (with J. Bok, N. Jedličková)

The *Wiener index* is a graph parameter originating from chemical graph theory. It is defined as the sum of the lengths of the shortest paths between all pairs of vertices in given graph. In 1991, Šoltés posed the following problem regarding Wiener index. Find all graphs such that its Wiener index is preserved upon removal of any vertex. The problem is far from being solved and to this day, only one such graph is known – the cycle graph on 11 vertices. In this paper we solve a relaxed version of the problem, proposed by Knor, Majstorović and Škrekovski. The problem is to find for a given k (infinitely many) graphs such that they have exactly k vertices such that if we remove any one of them, the Wiener index stays the same. We call such vertices *good* vertices and we show that there are infinitely many cactus graphs with exactly k cycles of length at least 7 that contain exactly $2k$ good vertices and infinitely many cactus graphs with exactly k cycles of length $c \in \{5, 6\}$ that contain exactly k good vertices. On the other hand, we prove that G has no good vertex if the length of the longest cycle in G is at most 4.

Asymmetric Ramsey properties of random graphs involving cliques and cycles

W. Mendonça (with A. Liebenau, L. Mattos, J. Skokan)

We prove that for every $\ell, r \geq 3$, there exists $c > 0$ such that for $p \leq cn^{-1/m_2(K_r, C_\ell)}$, with high probability there is a 2-edge-colouring of the random graph $\mathbf{G}_{n,p}$ with no monochromatic copy of K_r of the first colour and no monochromatic copy of C_ℓ of the second colour. This is a progress on a conjecture of Kohayakawa and Kreuter.

Covering 3-coloured random graphs with monochromatic trees

W. Mendonça (with Y. Kohayakawa, G. Mota, B. Schülke)

We investigate the problem of determining how many monochromatic trees are necessary to cover the vertices of an edge-coloured random graph. More precisely, we show that for $p \gg \left(\frac{\ln n}{n}\right)^{1/6}$ in any 3-colouring of the random graph $G(n,p)$ we can find 3 monochromatic trees such that their union covers all vertices. This improves, for three colours, a result of Bucić, Korándi and Sudakov [*Covering random graphs by monochromatic trees and Helly-type results for hypergraphs*, `arXiv:1902.05055`]

Exploring projective norm graphs

T. Mészáros (with T. Bayer, L. Rónyai, T. Szabó)

The projective norm graphs $\mathrm{NG}(q,t)$ provide tight constructions for the Turán number of complete bipartite graphs $K_{t,s}$ with $s > (t-1)!$. The determination of the largest integer s_t, such that the projective norm graph $\mathrm{NG}(q,t)$ contains K_{t,s_t} for all large enough prime powers q is an important open question with far-reaching general consequences. Here we settle the case $t = 4$. Along the way we also develop methods to count the copies of any fixed 3-degenerate subgraph, and find that projective norm graphs are quasirandom with respect to this parameter. Some of these results also extend the work of Alon and Shikhelman on generalized Turán numbers. Finally we also completely determine the automorphism group of $\mathrm{NG}(q,t)$ for every possible values of the parameters.

Mixing time of the swap Markov chain and *P*-stability

T. R. Mezei (with P. L. Erdős, C. S. Greenhill, I. Miklós, D. Soltész, L. Soukup)

The aim of this paper is to confirm that P-stability of a family of unconstrained/ bipartite/directed degree sequences is sufficient for the swap Markov chain to be rapidly mixing on members of the family. This is a common generalization of every known result that shows the rapid mixing nature of the swap Markov chain on a region of degree sequences. In addition, for example, it encompasses power-law degree sequences with exponent $\gamma > 2$, and, asymptotically almost surely, the degree sequence of any Erdős-Rényi random graph $G(n,p)$ where p is bounded away from 0 and 1 by at least $\frac{5\log n}{n-1}$. We also show that there exists a family of degree sequences which is not P-stable and its members have exponentially many realizations, yet the swap Markov chain is still rapidly mixing on them.

Splitting groups with cubic Cayley graphs of connectivity two

B. Miraftab (with K. Stavropoulos)

A group G splits over a subgroup C if G is either a free product with amalgamation $A \underset{C}{*} B$ or an HNN-extension $G = A \underset{C}{*} (t)$. We invoke tree-decompositions and Bass-Serre theory, and classify all infinite groups which admit cubic Cayley graphs of connectivity two in terms of splittings over a subgroup.

Cycles through a set of specified vertices of a planar graph

S. Mohr

Confirming a conjecture of Plummer, Thomas and Yu proved that a 4-connected planar graph contains a cycle through all but two (freely choosable) vertices. Here we prove that a planar graph G contains a cycle through $X \setminus \{x_1, x_2\}$ if $X \subseteq V(G)$, X large enough, $x_1, x_2 \in X$, and X cannot be separated in G by removing less than 4 vertices.

Testing isomorphism of circulant objects in polynomial time

M. Muzychuk (with I. Ponomarenko)

We show that isomorphism testing of two cyclic combinatorial objects may be done in a polynomial time provided that both objects share the same regular cyclic group of automorphisms given in advance.

Spreading linear triple systems and expander triple systems

Z. L. Nagy (with Z. L. Blázsik)

The existence of Steiner triple systems STS(n) of order n containing no nontrivial subsystem is well known for every admissible n. We generalize this result in two ways. First we define the expander property of 3-uniform hypergraphs and show the existence of Steiner triple systems which are almost perfect expanders. Next we define the strong and weak spreading property of linear hypergraphs, and determine the minimum size of a linear triple system with these properties, up to a small constant factor. A linear triple system on a vertex set V has the spreading, or respectively weakly spreading property if any sufficiently large subset $V' \subset V$ contains a pair of vertices with which a vertex of $V \setminus V'$ forms a triple of the system. Here the condition on V' refers to $|V'| \geq 4$ or V' is the support of more than one triples, respectively. This property is strongly connected to the connectivity of the structure the so-called influence maximisation. We also discuss how the results are related to Erdős' conjecture on locally sparse STSs, subsquare-free Latin-squares and possible applications in finite geometry.

On Heilbronn triangle-type problems in higher dimensions

K. Odermann (with F. S. Benevides, C. Hoppen, H. Lefmann)

The Heilbronn triangle problem is a classical geometrical problem that asks for a placement of n points in the unit-square $[0,1]^2$, that maximizes the smallest area of a triangle formed by those points. This problem has natural generalizations to higher dimensions. For integers $k, d \geq 2$ and a set $\mathcal{P}$ of n points in $[0,1]^d$, let $s = \min\{(k-1), d\}$ and $V_k^{(d)}(\mathcal{P})$ be the minimum s-dimensional volume of the convex hull of k points in $\mathcal{P}$. Here, instead of considering the supremum of $V_k^{(d)}(\mathcal{P})$, we consider its average value, $\widetilde{\Delta}_k^{(d)}(n)$, when the n points in $\mathcal{P}$ are chosen independently and uniformly at random in $[0,1]^d$. We prove that $\widetilde{\Delta}_k^{(d)}(n) = \Theta\left(n^{\frac{-k}{1+|d-k+1|}}\right)$, for every fixed $k, d \geq 2$.

Enumeration of unsensed orientable and non-orientable maps

A. Omelchenko (with E. Krasko)

The work is devoted to the problem of enumerating maps on an orientable or non-orientable surface of a given genus g up to all symmetries (so called unsensed maps). We obtain general formulas which reduce the problem of counting such maps to the problem of enumerating rooted quotient maps on orbifolds. In addition, we solve the problem of describing all cyclic orbifolds for a given orientable or non-orientable surface of fixed genus g. We also derive recurrence relations for quotient rooted maps on orbifolds that can be orientable or non-orientable surfaces with r branch points, h boundary components and g handles or cross-caps. These results allowed us to calculate the numbers of unsensed maps on orientable or non-orientable surfaces of arbitrary genus g by the number of edges.

Density and fractal property of the class of oriented trees

P. Oviedo (with J. Hubička, J. Nešetřil)

We show a density theorem for the class of finite proper trees ordered by the homomorphism order, where a proper tree is an oriented tree which is not homomorphic to a path. We also show that every interval of proper trees, in addition to being dense, is in fact universal. We end by considering the fractal property in the class of all finite digraphs. This complements the characterization of finite dualities of finite digraphs.

Independent transversals versus transversals

G. Paesani (with K. K. Dabrowski, M. Johnson, D. Paulusma, V. Zamaraev)

We compare the minimum size of a vertex cover, feedback vertex set and odd cycle transversal of a graph with the minimum size of the corresponding variants in which the transversal must be an independent set. We investigate for which graphs H the two sizes are equal whenever the graph in question belongs to the class of H-free graphs. We find complete classifications for vertex cover and almost complete classifications for feedback vertex set and odd cycle transversal.

Adaptive majority problems for restricted query graphs and for weighted sets

D. Pálvölgyi (with G. Damásdi, D. Gerbner, G. O. H. Katona, A. Methuku, B. Keszegh, D. Lenger, D. T. Nagy, B. Patkós, M. Vizer, G. Wiener)

Suppose that the vertices of a graph G are colored with two colors in an unknown way. The color that occurs on more than half of the vertices is called the *majority color* (if it exists), and any vertex of this color is called a *majority vertex*. We study the problem of finding a majority vertex (or show that none exists), if we can query edges to learn whether their endpoints have the same or different colors. Denote the least number of queries needed in the worst case by $m(G)$. It was shown by Saks and Werman that $m(K_n)=n-b(n)$ where $b(n)$ is the number of 1's in the binary representation of n. In this paper we initiate the study of the problem for general graphs. The obvious bounds for a connected graph G on n vertices are $n-b(n) \leq m(G) \leq n-1$. We show that for any tree T on an even number of vertices we have $m(T)=n-1$, and that for any tree T on an odd number of vertices, we have $n-65 \leq m(T) \leq n-2$. Our proof uses results about the weighted version of the problem for K_n, which may be of independent interest. We also exhibit a sequence G_n of graphs with $m(G_n)=n-b(n)$ such that the number of edges in G_n is $O(nb(n))$.

Almost spanning universality in random graphs

O. Parczyk

A graph G is called universal for a family of graphs $\mathcal{F}$ if it contains every element $F \in \mathcal{F}$ as a subgraph. We prove for $\Delta \geq 3$ and $\varepsilon > 0$ that $G(n,p)$ is a.a.s. universal for the family of all graphs on $(1-\varepsilon)n$ vertices with maximum degree Δ provided that $p=\omega(n^{-1/(\Delta-1)})$. This improves on previously known results by Conlon, Ferber, Nenadov, and Škorić [*Almost-spanning universality in random graphs*, Random Structures Algorithms **50** (2017), 380–393] and is asymptotically optimal for $\Delta=3$.

More non-bipartite forcing pairs

O. Parczyk (with T. Hubai, D. Kráľ, Y. Person)

We study pairs of graphs (H_1, H_2) such that every graph with the densities of H_1 and H_2 close to the densities of H_1 and H_2 in a random graph is quasirandom; such pairs (H_1, H_2) are called forcing. Non-bipartite forcing pairs were first discovered by Conlon, Hàn, Person and Schacht [*Weak quasi-randomness for uniform hypergraphs*, Random Structures Algorithms **40** (2012), no. 1, 1–38]: they showed that (K_t, F) is forcing where F is the graph that arises from K_t by iteratively doubling its vertices and edges in a prescribed way t times. Reiher and Schacht [*Forcing quasirandomness with triangles*, Forum of Mathematics, Sigma. Vol. 7, 2019] strengthened this result for $t=3$ by proving that two doublings suffice and asked for the minimum number of doublings needed for $t > 3$. We show that $\lceil (t+1)/2 \rceil$ doublings always suffice.

The size-Ramsey number of powers of bounded degree trees

O. Parczyk (with S. Berger, Y. Kohayakawa, G. S. Maesaka, T. Martins, W. Mendonça, G. O. Mota)

Given an integer $s \geq 1$, the *s-colour size-Ramsey number* of a graph H is the smallest integer m such that there exists a graph G with m edges with the property that, in any colouring of $E(G)$ with s colours, there is a monochromatic copy of H. We prove that, for any positive integers k and s, the s-colour size-Ramsey number of the kth power of any n-vertex bounded degree tree is linear in n.

Sharp bounds for decomposing graphs into edges and triangles

Y. Pehova (with A. Blumenthal, B. Lidický, O. Pikhurko, F. Pfender, J. Volec)

Let $\pi_3(G)$ be the minimum of twice the number of edges plus three times the number of triangles over all edge-decompositions of G into copies of K_2 and K_3. We are interested in the value of $\pi_3(n)$, the maximum of $\pi_3(G)$ over graphs G with n vertices. This specific extremal function was first studied by Gyori and Tuza [Decompositions of graphs into complete subgraphs of given order, Studia Sci. Math. Hungar. 22 (1987), 315–320], who showed that $\pi_3(n) \leq 9n^2/16$. In a recent advance on this problem, Král', Lidický, Martins and Pehova [arXiv:1710:08486] proved via flag algebras that $\pi_3(n) \leq (1/2 + o(1))n^2$, which is tight up to the $o(1)$ term. We extend their proof by giving the exact value of $\pi_3(n)$ for large n, and we show that K_n and $K_{\lfloor n/2 \rfloor, \lceil n/2 \rceil}$ are the only extremal examples.

On graphs with bounded induced odd cycle packing number

J. Pekárek (with Z. Dvořák)

We give several results on graphs avoiding the disjoint union of k odd cycles as an induced subgraph, including an improved polynomial-time approximation scheme for the independence number and an analysis of the relationship between the clique number and the chromatic number of these graphs.

A Turán-type theorem for large-distance graphs in Euclidean spaces, and related isodiametric problems

C. Pelekis (with M. Doležal, J. Hladký, J. Kolář, T. Mitsis, V. Vlasák)

A *large-distance graph* is a measurable graph whose vertex set is a measurable subset of $\mathbb{R}^d$, and two vertices are connected by an edge if and only if their distance is larger that 2. We address questions from extremal graph theory in the setting of large-distance graphs, focusing in particular on upper-bounds on the measures of vertices and edges of K_r-free large-distance graphs. Our main result states that if $A \subset \mathbb{R}^2$ is a measurable set such that the large-distance graph on A does not contain any complete subgraph on three vertices then the 2-dimensional Lebesgue measure of A is at most 2π.

On unit grid intersection graphs and several other intersection graph classes

M. Pergel (with I. Mustață)

We explore what could make recognition of particular intersection-defined classes hard. We focus mainly on unit grid intersection graphs (UGIGs), i.e., intersection graphs of unit-length axis-aligned segments and grid intersection graphs (GIGs, which are defined like UGIGs without unit-length restriction). As side effects we obtain several further nontrivial results. We show that the explored graph classes are NP-hard to recognized even when restricted to graphs with arbitrarily large girth, i.e., length of a shortest cycle. Next we show that the recognition of these classes remains hard even for graphs with restricted degree (4, 5 and 8 depending on a particular class). For UGIGs we present structural results on the size of a possible representation, too.

Recognising the overlap graphs of subtrees of restricted trees is hard

M. Pergel (with J. Enright)

The overlap graphs of subtrees in a tree (SOGs) generalise many other graphs classes with set representation characterisations. The complexity of recognising SOGs in open. The complexities of recognising many subclasses of SOGs are known. We consider several subclasses of SOGs by restricting the underlying tree. For a fixed integer $k \geq 3$, we consider:

- The overlap graphs of subtrees in a tree where that tree has k leaves.
- The overlap graphs of subtrees in trees that can be derived from a given input tree by subdivision and have at least three leaves.
- The overlap and intersection graphs of paths in a tree where that tree has maximum degree k.

We show that the recognition problems of these classes are NP-complete. For all other parameters we get circle graphs, well known to be polynomially recognizable.

A Brooks-like result for graph powers

T. Pierron

Coloring a graph G consists in finding an assignment of colors c $V(G) \to \{1,\ldots,p\}$ such that any pair of adjacent vertices receive different colors. The minimum integer p such that a coloring exists is called the chromatic number of G, denoted by $\chi(G)$. We investigate the chromatic number of powers of graphs, i.e. the graphs obtained from a graph G by adding an edge between every pair of vertices at distance at most k. For $k = 1$, Brooks' theorem states that every graph of maximum degree $\Delta \geq 3$ excepted the clique on $\Delta + 1$ vertices can be colored using Δ colors (i.e., one color less than the naive upper bound). For $k \geq 2$, a similar result holds: excepted for Moore graphs, the naive upper bound can be lowered by 2. We prove that for $k \geq 3$ and for every Δ, we can actually spare $k - 2$ colors, excepted for a finite number of graphs.

Localised codegree conditions for tight Hamiltonian cycles in 3-uniform hypergraphs

S. Piga (with P. Araújo, M. Schacht)

We study sufficient conditions for the existence of Hamiltonian cycles in uniformly dense 3-uniform hypergraphs. Problems of this type were first considered by Lenz, Mubayi, and Mycroft for loose Hamiltonian cycles and Aigner-Horev and Levy considered it for tight Hamiltonian cycles for a fairly strong notion of uniformly dense hypergraphs. We focus on tight cycles and obtain optimal results for a weaker notion of uniformly dense hypergraphs. We show that if an n-vertex 3-uniform hypergraph $H = (V, E)$ has the property that for any set of vertices X and for any collection P of pairs of vertices, the number of hyperedges composed by a pair belonging to P and one vertex from X is at least $(1/4 + o(1))|X||P| - o(|V|^3)$ and H has minimum vertex degree at least $\Omega(|V|^2)$, then H contains a tight Hamiltonian cycle. A probabilistic construction shows that the constant $1/4$ is optimal in this context.

The Kuperberg conjecture for translates of convex bodies

R. Prosanov

We prove that if a convex body C admits a dense translative packing, then it admits an economical translative covering and vice versa. This answers positively to the question of W. Kuperberg in the case of translative arrangements.

Acyclic improper choosability of subcubic graphs

A. Raspaud (with M. Chen)

A d-improper k-coloring of a graph G is a mapping $\varphi : V(G) \to \{1, 2, \ldots, k\}$ such that for every color i, the subgraph induced by the vertices of color i has maximum degree d. That is, every vertex can be adjacent to at most d vertices with being the same color as itself. Such a d-improper k-coloring is further said to be acyclic if for every pair of distinct colors, say i and j, the induced subgraph by the edges whose endpoints are colored with i and j is a forest. Meanwhile, we say that G is acyclically $(k, d)^*$-colorable. A graph G is called acyclically d-improper L-colorable if for a given list assignment $L = \{L(v) \mid v \in V(G)\}$, there exists an acyclic d-improper coloring φ such that $\varphi(v) \in L(v)$ for each vertex v. If G is acyclically d-improper L-colorable for any list assignment L with $|L(v)| \geq k$ for all $v \in V$, then we say that G is acyclically d-improper k-choosable, or simply say that G is acyclically $(k, d)^*$-choosable. It is known that every subcubic graph is acyclically $(2, 2)^*$-colorable. But there exists a 3-regular graph that is not necessarily acyclically $(2, 2)^*$-choosable. We present our following result: every non-3-regular subcubic graph is acyclically $(2, 2)^*$-choosable.

A graphon perspective for fractional isomorphism

I. Rocha (with J. Grebík)

Fractional isomorphism of graphs plays an important role in practical applications of graph isomorphism test by means of the color refinement algorithm. We introduce a suitable generalization to the space of graphons in terms of Markov opertors on a Hilbert space, provide characterizations in terms of a push-forward of the graphon to a quotient space and also in terms of measurable partitions of the underlying space. Our proofs use a weak version of the mean ergodic theorem, and correspondences between objects such as Markov projections, sub-σ-algebras, measurable decompositions, etc. That also provides an alternative proof for the characterizations of fractional isomorphism of graphs without the use of Birkhoff–von Neumann Theorem.

On a Frankl-Wilson theorem and its geometric corollaries

A. A. Sagdeev (with A. M. Raigorodskii)

We find a new analogue of the Frankl-Wilson theorem on the independence number of distance graphs of some special type. We apply this new result to two combinatorial geometry problems. First, we improve a previously known value c such that $\chi(\mathbb{R}^n; S_2) \geq (c + o(1))^n$, where $\chi(\mathbb{R}^n; S_2)$ is the minimum number of colors needed to color all points of $\mathbb{R}^n$ so that there is no monochromatic set of vertices of a unit equilateral triangle S_2. Second, given $m \geq 3$ we improve the value ξ_m such that for any $n \in \mathbb{N}$ there is a distance graph in $\mathbb{R}^n$ with the girth greater than m and the chromatic number at least $(\xi_m + o(1))^n$.

On some extremal results for order types

M. T. Sales (with J. Han, Y. Kohayakawa, H. Stagni)

A *configuration* is a finite set of points in the plane. Two configurations A and B have the same *order type* if there exists a bijection between them preserving the orientation of every ordered triple. We investigate the following extremal problem on embedding configurations in general position in integer grid. Given an order type B, let $\mathrm{ex}(N, B)$ be the maximum integer m such that there exists a subconfiguration of the integer grid $[N]^2$ of size m without a copy of B. An application of the celebrated multidimensional Szemerédi's theorem gives $\mathrm{ex}(N, B) = o(N^2)$. We first prove a subquadratic upper bound for all large order types B and large N, namely, $\mathrm{ex}(N, B) \leq N^{2-\eta}$ for some $\eta = \eta(B) > 0$. Then we give improved bounds for specific order types: we show that $\mathrm{ex}(N, B) = O(N)$ for the convex order type B, and $\mathrm{ex}(N, B) = N^{3/2+o(1)}$ for those B satisfying the so-called Erdős-Hajnal property. Our approach is to study the inverse problem, that is, the smallest $N_0 = N_0(\alpha, B)$ such that every α proportion of $[N_0]^2$ contains a copy of B.

The structure of hypergraphs without long Berge cycles

N. Salia (with E. Győri, N. Lemons, O. Zamora)

We study the structure of r-uniform hypergraphs containing no Berge cycles of length at least k for $k \leq r$, and determine that such hypergraphs have some special substructure. In particular we determine the extremal number of such hypergraphs, giving an affirmative answer to the conjectured value when $k = r$ and giving a a simple solution to a recent result of Kostochka-Luo when $k < r$.

Gallai's path decomposition conjecture for graphs with maximum E-degree at most 3

M. Sambinelli (with F. Botler)

A path decomposition of a graph G is a collection of edge-disjoint paths of G that covers the edge set of G. Gallai (1968) conjectured that every connected graph on n vertices admits a path decomposition of cardinality at most $\lfloor (n+1)/2 \rfloor$. Seminal results toward its verification consider the graph obtained from G by removing its vertices with odd degree, which is called the *E-subgraph* of G. Lovász (1968) verified Gallai's Conjecture for graphs whose E-subgraphs consist of at most one vertex, and Pyber (1996) verified it for graphs whose E-subgraphs are forests. In 2005, Fan verified Gallai's Conjecture for graphs whose E-subgraphs are triangle-free and contain only blocks with maximum degree at most 3. Since then, no result was obtained regarding E-subgraphs. In this paper, we verify Gallai's Conjecture for graphs whose E-subgraphs have maximum degree at most 3.

On disjoint holes in point sets

M. Scheucher

Given a set of points $S \subseteq \mathbb{R}^2$, a subset $X \subseteq S$, $|X| = k$, is called *k-gon* if all points of X lie on the boundary of conv(X), and *k-hole* if, in addition, no point of $S \setminus X$ lies in conv(X). We use computer assistance to show that every set of 17 points in general position admits two *disjoint* 5-holes, that is, holes with disjoint respective convex hulls. This answers a question of Hosono and Urabe (2001). In a recent article, Hosono and Urabe (2018) present new results on interior-disjoint holes – a variant, which also has been investigated in the last two decades. Using our program, we show that every set of 15 points contains two interior-disjoint 5-holes. Moreover, our program can be used to verify that every set of 17 points contains a 6-gon within significantly smaller computation time than the original program by Szekeres and Peters (2006).

On orthogonal symmetric chain decompositions

M. Scheucher (with K. Däubel, S. Jäger, T. Mütze)

The *n-cube* is the poset obtained by ordering all subsets of $\{1,\dots,n\}$ by inclusion, and it can be partitioned into $\binom{n}{\lfloor n/2 \rfloor}$ chains, which is the minimum possible number. Two such decompositions of the n-cube are called *orthogonal* if any two chains of the decompositions share at most a single element. Shearer and Kleitman conjectured in 1979 that the n-cube has $\lfloor n/2 \rfloor + 1$ pairwise orthogonal decompositions into the minimum number of chains, and they constructed two such decompositions. Spink recently improved this by showing that the n-cube has three pairwise orthogonal chain decompositions for $n \geq 24$. In this paper, we construct four pairwise orthogonal chain decompositions of the n-cube for $n \geq 60$. We also construct five pairwise *edge-disjoint* symmetric chain decompositions of the n-cube for $n \geq 90$, where edge-disjointness is a slightly weaker notion than orthogonality, improving on a recent result by Gregor, Jäger, Mütze, Sawada, and Wille.

Minimum pair-degree for tight Hamiltonian cycles in 4-uniform hypergraphs

B. Schülke (with C. Reiher, V. Rödl, A. Ruciński, M. Schacht)

We show that every 4-uniform hypergraph with n vertices and minimum pair-degree at least $(5/9 + o(1))n^2/2$ contains a tight Hamiltonian cycle. This degree condition is asymptotically optimal. In the proof we use a variant of the absorbing method and ideas from the proof of the optimal minimum vertex degree condition for tight Hamiltonian cycles in 3-uniform hypergraphs that was obtained in a previous work by Reiher, Rödl, Ruciński, Schacht, and Szemerédi.

Deviation probabilities for arithmetic progressions and other regular discrete structures

O. Serra (with G. Fiz Pontiveros, S. Griffiths, M. Secco)

Let $\mathcal{H}$ be a k-uniform hypergraph on a vertex set V and B_m be a uniformly sampled m-set from V. Set X to be the random variable given by the number of edges induced by the set B_m. We provide tight upperbounds (up to a constant in the exponent) for the tail distribution of $X - \mathbb{E}(X)$ for a wide range of deviations, provided some near regularity conditions are satisfied by the hypergraph $\mathcal{H}$. In particular, the bounds may be applied to the setting of arithmetic progressions and more generally to solutions of linear systems.

Two values of the chromatic number of a sparse random graph

D. Shabanov (with S. Kargaltsev, T. Shaikheeva)

The famous results of Łuczak (1991) and Alon – Krivelevich (1997) state that the chromatic number $\chi(G(n,p))$ of the binomial random graph $G(n,p)$ is concentrated in two consecutive values with probability tending to 1 provided $p = p(n) \leq n^{-1/2-\varepsilon}$. Unfortunately, their proofs do not give the explicit values of $\chi(G(n,p))$ as functions of n and p. Achlioptas and Naor (2005) found these values in the sparse case when np is fixed. Coja-Oghlan, Panagiotou and Steger (2008) showed that the chromatic number of $G(n,p)$ is concentrated in three explicit consecutive values provided $p = p(n) \leq n^{-3/4-\delta}$, they also established a 2-point concentration for the "half" of the values of the parameter p under these conditions. In the current paper we improve the discussed result and show that the concentration of the chromatic number in two explicit consecutive values holds "almost everywhere" provided $p = p(n) \leq n^{-3/4-\delta}$ and $np \to +\infty$. Namely, if $r_p = \min\{r : (n-1)p < 2r \ln r\}$ then we prove that for

$$(n-1)p \in \left(2(r_p-1)\ln(r_p-1), 2r_p \ln r_p - \ln r_p - 2 - r_p^{-1/6}\right),$$

it holds that

$$\mathsf{Pr}\left(\chi(G(n,p)) \in \{r_p, r_p+1\}\right) \to 1 \text{ as } n \to +\infty.$$

The asymptotics of reflectable weighted walks in arbitrary dimension

S. Simon (with M. Mishna)

We consider the weighted lattice walks with a reflectable step set restricted to the positive d-dimensional orthant. We obtain asymptotic formulas for the number of such walks as a function of the weights. To do so, we set up the desired generating function as the diagonal of a rational function. Then we perform a coefficient extraction via an integral computation which is broken up into two cases. One part uses the residue theorem to evaluate the integral within an error, while the other uses known approximations of Fourier-Laplace integrals.

On the density of C_7-critical graphs

E. Smith-Roberge (with L. Postle)

In 1959, Grötszch famously proved that every planar graph of girth at least 4 admits a homomorphism to C_3. A natural generalization is the following conjecture: for every positive integer t, every planar graph of girth at least $4t$ admits a homomorphism to C_{2t+1}. This is the planar dual of a well-known conjecture of Jaeger, which states that every $4t$-edge-connected graph admits a modulo $(2t+1)$-orientation. Though Jaeger's original conjecture was recently disproved, it has been shown to hold for $6t$-edge-connected graphs. This implies that every planar graph of girth at least $6t$ admits a homomorphism to C_{2t+1}. We improve upon the $t = 3$ case, by showing that every planar graph of girth at least 16 admits a homomorphism to C_7. We obtain this through a more general result regarding the density of critical graphs: if G is a C_7-critical graph with $G \notin \{C_3, C_5\}$, then $e(G) \geq \frac{17v(G)-2}{15}$. Our girth bound is the best known result for Jaeger's Conjecture in the $t = 3$ case.

The evolution of random graphs on surfaces of non-constant genus

P. Sprüssel (with C. Dowden, M. Kang, M. Moßhammer)

Given a graph G, the *genus* of G denotes the smallest integer g for which G can be drawn on the orientable surface of genus g without crossing edges. For integers $g, m \geq 0$ and $n > 0$, we let $S_g(n, m)$ denote the graph taken uniformly at random from the set of all graphs on $\{1, 2, \ldots, n\}$ with exactly $m = m(n)$ edges and with genus at most $g = g(n)$. We investigate the evolution of $S_g(n, m)$ as m increases, focussing on the number $|H_1|$ of vertices in the largest component. For $g \ll n$, we show that $|H_1|$ exhibits two phase transitions, one at around $m = \frac{n}{2}$ and a second one at around $m = n$. The exact behaviour of $|H_1|$ in the critical windows of these phase transitions depends on the order of $g = g(n)$.

Colouring non-even digraphs

R. Steiner (with M. G. Millani, S. Wiederrecht)

A colouring of a digraph as defined by Neumann-Lara in 1982 is a vertex-colouring such that no monochromatic directed cycle exists. The minimal number of colours required for such a colouring of a digraph is defined to be its *dichromatic number*. This quantity has been widely studied in the last decades and is a natural directed analogue of the chromatic number of a graph. A digraph D is called *even* if for every 0-1-weighting of the edges it contains a directed cycle of even total weight. We show that every *non-even* digraph has dichromatic number at most 2 and an optimal colouring can be found in polynomial time. We show that every *non-even* digraph has dichromatic number at most 2 and an optimal colouring can be found in polynomial time. We strengthen a previously known NP-hardness result by showing that deciding whether a directed graph is 2-colourable remains NP-hard even if it contains a feedback vertex set of bounded size.

Coset geometries with trialities and their reduced incidence graphs

K. Stokes (with D. Leemans)

In this article we explore combinatorial trialities of incidence geometries. We give a construction that uses coset geometries to construct examples of incidence geometries with trialities and prescribed automorphism group. We define the reduced incidence graph of the geometry to be the oriented graph obtained as the quotient of the geometry under the triality. Our chosen examples exhibit interesting features relating the automorphism group of the geometry and the automorphism group of the reduced incidence graphs.

Ordered graphs and large bi-cliques in intersection graphs of curves

I. Tomon (with J. Pach)

An *ordered graph* $G_<$ is a graph with a total ordering $<$ on its vertex set. A *monotone path* of length k is a sequence of vertices $v_1 < v_2 < \cdots < v_k$ such that v_iv_j is an edge of $G_<$ if and only if $|j-i| = 1$. A *bi-clique* of size m is a complete bipartite graph whose vertex classes are of size m. We prove that for every positive integer k, there exists a constant $c_k > 0$ such that every ordered graph on n vertices that does not contain a monotone path of length k as an induced subgraph has a vertex of degree at least c_kn, or its complement has a bi-clique of size at least $c_kn/\log n$. A similar result holds for ordered graphs containing no induced ordered subgraph isomorphic to a fixed ordered matching. As a consequence, we give a short combinatorial proof of the following theorem of Fox and Pach. There exists a constant $c > 0$ such the intersection graph G of any collection of n x-monotone curves in the plane has a bi-clique of size at least $cn/\log n$ or its complement contains a bi-clique of size at least cn. (A curve is called x-monotone if every vertical line intersects it in at most one point.) We also prove that if G has at most $\left(\frac{1}{4}-\epsilon\right)\binom{n}{2}$ edges for some $\epsilon > 0$, then $\overline{G}$ contains a linear sized bi-clique. We show that this statement does not remain true if we replace $\frac{1}{4}$ by any larger constants.

The maximum number of P_ℓ copies in P_k-free graphs

C. Tompkins (with E. Győri, N. Salia, O. Zamora)

Generalizing Turán's classical extremal problem, Alon and Shikhelman investigated the problem of maximizing the number of copies of T in an H-free graph, for a pair of graphs T and H. Whereas Alon and Shikhelman were primarily interested in determining the order of magnitude for some classes of graphs H, we focus on the case when T and H are paths, where we find asymptotic and exact results in some cases. We also consider other structures like stars and the set of cycles of length at least k, where we derive asymptotically sharp estimates. Our results generalize well-known extremal theorems of Erdős and Gallai.

Minor-Obstructions for Apex Sub-unicyclic Graphs

V. Velona (with A. Leivaditis, A. Singh, G. Stamoulis, D. M. Thilikos, K. Tsatsanis)

A graph is *sub-unicyclic* if it contains at most one cycle. We also say that a graph G is *k-apex sub-unicyclic* if it can become sub-unicyclic by removing k of its vertices. We identify 29 graphs that are the minor-obstructions of the class of 1-apex sub-unicyclic graphs, i.e., the set of all minor minimal graphs that do not belong in this class. For bigger values of k, we give an exact structural characterization of all the cactus graphs that are minor-obstructions of k-apex sub-unicyclic graphs and we enumerate them. This implies that, for every k, the class of k-apex sub-unicyclic graphs has at least $0.34 \cdot k^{-2.5}(6.278)^k$ minor-obstructions.

Extremal families for Kruskal-Katona Theorem

L. Vena (with O. Serra)

Given a set of size n and a positive integer $k < n$, Kruskal-Katona theorem gives the minimum size of the shadow of a family S of k-sets of $[n]$ in terms of the cardinality of S. We give a characterization of the families of k-sets satisfying equality in Kruskal-Katona theorem. This answers a question of Füredi and Griggs.

The canonical Tutte polynomial for signed graphs

L. Vena (with A. Goodall, B. Litjens, G. Regts)

We construct a new polynomial invariant for signed graphs, the trivariate Tutte polynomial, which contains among its evaluations the number of proper colorings and the number of nowhere-zero flows. In this, it parallels the Tutte polynomial of a graph, which contains the chromatic polynomial and flow polynomial as specializations. While the Tutte polynomial of a graph is equivalently defined as the dichromatic polynomial or Whitney rank polynomial, the dichromatic polynomial of a signed graph (defined more widely for biased graphs by Zaslavsky) does not, by contrast, give the number of nowhere-zero flows as an evaluation in general. The trivariate Tutte polynomial contains Zaslavsky's dichromatic polynomial as a specialization. Furthermore, the trivariate Tutte polynomial gives as an evaluation the number of proper colorings of a signed graph under a more general sense of signed graph coloring in which colors are elements of an arbitrary finite set equipped with an involution.

Generalized Turán problems for even cycles

M. Vizer (with D. Gerbner, E. Győri, A. Methuku)

Given a graph H and a set of graphs $\mathcal{F}$, let $\mathrm{ex}(n, H, \mathcal{F})$ denote the maximum possible number of copies of H in an $\mathcal{F}$-free graph on n vertices. We investigate the function $\mathrm{ex}(n, H, \mathcal{F})$, when H and members of $\mathcal{F}$ are cycles. Let C_k denote the cycle of length k and let $\mathscr{C}_k = \{C_3, C_4, \dots, C_k\}$. We highlight the main results below.

(i) We show that $\mathrm{ex}(n, C_{2l}, C_{2k}) = \Theta(n^l)$ for any $l, k \geq 2$. Moreover, in some cases we determine it asymptotically.

(ii) Erdős's Girth Conjecture states that for any positive integer k, there exist a constant $c > 0$ depending only on k, and a family of graphs $\{G_n\}$ such that $|V(G_n)| = n$, $|E(G_n)| \geq cn^{1+1/k}$ with girth more than $2k$. Solymosi and Wong proved that if this conjecture holds, then for any $l \geq 3$ we have $ex(n, C_{2l}, \mathscr{C}_{2l-1}) = \Theta(n^{2l/(l-1)})$. We prove that their result is sharp in the sense that forbidding any other even cycle decreases the number of C_{2l}'s significantly.

(iii) We prove $\mathrm{ex}(n, C_{2l+1}, \mathscr{C}_{2l}) = \Theta(n^{2+1/l})$, provided a stronger version of Erdős's Girth Conjecture holds (which is known to be true when $l = 2, 3, 5$). This result is also sharp in the sense that forbidding one more cycle decreases the number of C_{2l+1}'s significantly.

On Ramsey and Turán problems of edge-ordered graphs

M. Vizer (with M. Balko, D. Gerbner, A. Methuku, D. T. Nagy, D. Pálvölgyi, G. Tardos)

We introduce the Turán problem for edge ordered graphs. We call a simple graph *edge ordered*, if its edges are linearly ordered. An isomorphism between edge ordered graphs must respect the edge order. A subgraph of an edge ordered graph is itself an edge ordered graph with the induced edge order. We say that an edge ordered graph G *avoids* another edge ordered graph H, if no subgraph of G is isomorphic to H. The Turán number $\mathrm{ex}'_<(n, \mathcal{H})$ of a family $\mathcal{H}$ of edge ordered graphs is the maximum number of edges in an edge ordered graph on n vertices that avoids all elements of $\mathcal{H}$. We examine this parameter in general and also for several singleton families of edge orders of certain small specific graphs, like star forests, short paths and the cycle of length four.

We introduce and study a variant of Ramsey numbers for edge-ordered graphs. The *edge-ordered Ramsey number* $\overline{R}_e(\mathfrak{G})$ of an edge-ordered graph $\mathfrak{G}$ is the minimum positive integer N such that there exists an edge-ordered complete graph $\mathfrak{K}_N$ on N vertices such that every 2-coloring of the edges of $\mathfrak{K}_N$ contains a monochromatic copy of $\mathfrak{G}$ as an edge-ordered subgraph of $\mathfrak{K}_N$. We prove that the edge-ordered Ramsey number $\overline{R}_e(\mathfrak{G})$ is finite for every edge-ordered graph $\mathfrak{G}$ and we obtain better estimates for special classes of edge-ordered graphs. In particular, we prove $\overline{R}_e(\mathfrak{G}) \leq 2^{O(n^3 \log n)}$ for every bipartite edge-ordered graph $\mathfrak{G}$ on n vertices. We also introduce a natural class of edge-orderings, called *lexicographic edge-orderings*, for which we can prove much better upper bounds on the corresponding edge-ordered Ramsey numbers.

Degree conditions forcing oriented cycles

J. Volec (with R. Glebov, A. Grzesik)

The longstanding Caccetta-Häggkvist Conjecture is asking for the minimum outdegree (or semidegree) in an oriented graph that forces the appearance of a directed cycle of a bounded length. Motivated by this, Kelly, Kühn and Osthus made a conjecture on the minimal semidegree forcing the appearance of a directed cycle of a given length, and proved it for cycles of length not divisible by 3. Here we prove all the remaining cases of their conjecture with the optimal semidegree threshold.

Coloring triangle-free L-graphs with $O(\log \log n)$ colors

B. Walczak

It is proved that triangle-free intersection graphs of n L-shapes in the plane have chromatic number $O(\log \log n)$. This improves the previous bound of $O(\log n)$ (McGuinness, 1996) and matches the known lower bound construction (Pawlik et al., 2013).

Ramsey numbers of Berge-hypergraphs and related structures

O. Zamora Luna (with N. Salia, C. Tompkins, Z. Wang)

For a graph $G = (V, E)$, a hypergraph $\mathcal{H}$ is called a *Berge-G*, denoted by BG, if there exists an injection $f\ E(G) \to E(\mathcal{H})$ such that for every $e \in E(G)$, $e \subseteq f(e)$. Let the Ramsey number $R^r(BG, BG)$ be the smallest integer n such that for any 2-edge-coloring of a complete r-uniform hypergraph on n vertices, there is a monochromatic Berge-G subhypergraph. In this paper, we show that the 2-color Ramsey number of Berge cliques is linear. In particular, we show that $R^3(BK_s, BK_t) = s + t - 3$ for $s, t \geq 4$ and $\max(s, t) \geq 5$ where BK_n is a Berge-K_n hypergraph. We also investigate the Ramsey number of trace hypergraphs, suspension hypergraphs and expansion hypergraphs.

On the Chromatic Index of Complementary Prisms

L. M. Zatesko (with R. Carmo, A. L. P. Guedes, A. Zorzi, R. C. S. Machado, C. M. H. Figueiredo)

This paper addresses the edge-colouring problem restricted to the graph class of complementary prisms. This graph class includes the Petersen graph, a very important and widely studied graph in the context of graph edge-colouring and remarkable related open questions, such as the Overfull Conjecture. We prove that all non-regular complementary prisms are *Class 1* and we conjecture that the only *Class 2* regular complementary prism is the Petersen graph. We present evidences for this conjecture.

Switches in Eulerian graphs

A. N. Zehmakan (with J. Nummenpalo, A. Pilz, D. Wolleb-Graf)

We show that the graph transformation problem of turning a simple graph into an Eulerian one by a minimum number of single edge switches is NP-hard. Further, we show that any simple Eulerian graph can be transformed into any other such graph by a sequence of 2-switches (i.e., exchange of two edge pairs), such that every intermediate graph is also Eulerian. However, finding the shortest such sequence also turns out to be an NP-hard problem.

Target set in threshold models

A. N. Zehmakan

Consider a graph G and an initial coloring, where each node is blue or red. In each round, all nodes simultaneously update their color based on a predefined rule. In a threshold model, a node becomes blue if a certain number or fraction of its neighbors are blue and red otherwise. What is the minimum number of nodes which must be blue initially so that the whole graph becomes blue eventually? We study this question for graphs which have expansion properties, parameterized by spectral gap, in particular the Erdős–Rényi random graph and random regular graphs.

Length of Cycles in Generalized Petersen Graphs

Z.-B. Zhang (with Z. Chen)

There have been extensive researchs on cycles in regular graphs, particularly 3-connected cubic graphs. Generalized Petersen graphs, denoted by $GP(n, k)$, are highly symmetric 3-connected cubic graphs, which have attracted great attention. The Hamiltonicity of

$GP(n,k)$ has been studied for a long time and thoroughly settled. Inspired by Bondy's meta-conjecture that almost every nontrivial condition for Hamiltonicity also implies pancyclicity, we seek for more cycle structures in this class of graphs, by figuring out the possible lengths of cycles in them. It turns out that generalized Petersen graphs, though not generally pancyclic, miss only very few possible length of cycles. For $k \in \{2, 3\}$, we completely determine all possible cycle lengths in $GP(n,k)$. We also obtain some results for $GP(n,k)$ where k is odd. In particular, when k is odd, and n is even and sufficiently large, $GP(n,k)$ is bipartite and weakly even pancyclic.

Maximum induced subgraphs of the binomial random graph

M. Zhukovskii (with J. Balogh)

We prove that a.a.s. the maximum size of an induced tree in the binomial random graph $G(n,p)$ is concentrated in four consecutive points. We also consider the following problem. Given $e(k)$, what is the maximum k such that $G(n,p)$ has an induced subgraph with k vertices and $e(k)$ edges? For $e = o(\frac{k \ln k}{\ln \ln k})$, we prove that a.a.s. this maximum size is concentrated in two consecutive points. In contrast, for $e(k) = p\binom{k}{2} + O(k)$, we prove that this size is not concentrated in any finite set. Moreover, we prove that for an $\omega_n \to \infty$, a.a.s. the size of the concentration set is smaller than $\omega_n\sqrt{n/\ln n}$. Otherwise, for an arbitrary constant $C > 0$, a.a.s. it is bigger than $C\sqrt{n/\ln n}$.

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t-strong cliques and the degree-diameter problem

M. Śleszyńska-Nowak (with M. Dębski)

For a graph G, $L(G)^t$ is the t-th power of the line graph of G – that is, vertices of $L(G)^t$ are edges of G and two edges $e, f \in E(G)$ are adjacent in $L(G)^t$ if G contains a path with at most t vertices that starts in a vertex of e and ends in a vertex of f. The *t-strong chromatic index* of G is the chromatic number of $L(G)^t$ and a *t-strong clique* in G is a clique in $L(G)^t$. Finding upper bounds for the *t-strong chromatic index* and *t-strong clique* are problems related to two famous problems: the conjecture of Erdős and Nešetřil concerning the strong chromatic index and the degree/diameter problem. We prove that the size of a t-strong clique in a graph with maximum degree Δ is at most $1.75\Delta^t + O\left(\Delta^{t-1}\right)$, and for bipartite graphs the upper bound is at most $\Delta^t + O\left(\Delta^{t-1}\right)$. We also show results for some special classes of graphs: $K_{1,r}$-free graphs and graphs with a large girth.

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